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基于时滞分割技术的时滞神经网络系统时滞相依全局稳定性分析

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摘 要:通过构造一个新的增广Lyapunov-Krasovskii泛函,利用时滞分割技术并结合自由权矩阵、Jensen积分不等式,得到一个时滞神经网络系统时滞相依全局渐近稳定新判据。该判据以LMI的形式给出,便于计算和验证。数值实例表明,文章结果改进了相关文献结论,具有更低的保守性。

关键词:时滞神经网络系统(DNN);全局渐近稳定;时滞分割技术;自由权矩阵;Jensen积分不等式

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近年来,由于在图像处理,模式识别,联想记忆及优化问题等方面的潜在应用,神经网络系统(NNs)受到众多学者的广泛关注和研究[1-4]。众所周知,神经网络系统的诸多应用极大地依赖于其动力学行为,尤其是其平衡点的存在性和稳定性。而且,一方面由于放大器有限的转换速度和有限的信息处理速度导致时滞在神经网络系统中往往是不可避免的,甚至成为系统不稳定或产生震荡的一个重要根源。另一方面,有的神将网络系统可能并不具备人们所需要的动力学行为。在无时滞神经网络系统中引入恰当的时滞成为解决这些问题的方法之一,例如,Yang和Cao就将时滞映射神将网络系统用于求解二次规划[5],Li将时滞神经网络系统用于求解凸规划并指出若能选择引入恰当的时滞,则可获得凸规划精确的最优解[6]。时滞神经网络系统的稳定性无论是在理论上还是在实践中都具有重要的意义。因此,关于时滞神经网络系统大量的稳定性研究成果,不管是时滞相依的,还是时滞独立的不断被提出[7-28]。由于含有时滞的相关信息,一般而言,时滞相依的稳定性条件要比时滞独立的稳定性条件具有更低的保守性,尤其是对于小时滞神经网络系统更是如此。于是,学者们更多地关注于寻求能使得时滞神经网络系统保持全局渐近或指数稳定的时滞最大允许上界(MAUB),大量具有更低保守性的稳定性条件被提出。然而,正如很多学者指出的,传统方式构造的Lyapunov-Krasovskii泛函对系统稳定性条件的保守性的降低所起作用已很小[14,16-17];而最近,时滞分割技术则被有效应用于降低系统稳定性条件的保守性[18-28]。

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鉴于此,本文将构造一个新的Lyapunov-Krasovskii泛函,利用时滞分割技术并结合使用自由权矩阵和Jensen积分不等式,更精细地估计Lyapunov-Krasovskii泛函导数的上界,以获取由线性矩阵不等式(LMI)表达的具有更低保守性的系统时滞相依全局渐近稳定性条件。在本文中,$\mathbb{R}^{n\times m}$ 表示 $n\times m$ 实矩阵空间,上标T表示转置,$\boldsymbol{X}\geqslant\boldsymbol{Y}$ ($\boldsymbol{X}>\boldsymbol{Y}$)表示矩阵 $\boldsymbol{X}-\boldsymbol{Y}$ 半正定(或正定),其中的 $\boldsymbol{X}$ 和 $\boldsymbol{Y}$ 都是对称阵,$\boldsymbol{I}_{n\times n}$ 、$\boldsymbol{O}_{n\times n}$ 分别表示 $n\times n$ 维的单位阵和零矩阵,记号 $*$ 总用于表示对称矩阵以及 $\mathrm{sym}(\boldsymbol{A})=\boldsymbol{A}+\boldsymbol{A}^{\mathrm{T}}$ 中的对称块。

1 问题描述

考虑如下的定常时滞神经网络模型:

$$\dot{\boldsymbol{y}}(t)=-\boldsymbol{C}\boldsymbol{y}(\boldsymbol{t})+\boldsymbol{A}\boldsymbol{g}(\boldsymbol{y}(\boldsymbol{t}))+\boldsymbol{B}\boldsymbol{g}(\boldsymbol{y}(\boldsymbol{t}-\tau))+\boldsymbol{I}\text{。} \quad (1)$$

式(1)中:$\boldsymbol{g}(\boldsymbol{y}(\boldsymbol{t}))=\left(g_1(y_1(t)),g_2(y_2(t)),\cdots,g_n(y_n(t))\right)^{\mathrm{T}}$ 、$\boldsymbol{y}(\boldsymbol{t})=\left(y_1(t),y_2(t),\cdots,y_n(t)\right)^{\mathrm{T}}$ 分别表示神经元激励函数和神经元状态向量;$\boldsymbol{C}=\mathrm{diag}(c_1,c_2,\cdots,c_n)$ 为神经元自反馈系数矩阵;$\boldsymbol{A},\boldsymbol{B}\in\mathbb{R}^{n\times n}$ 分别表示连接权矩阵和时滞连接权矩阵;$\boldsymbol{I}=\mathrm{diag}(I_1,I_2,\cdots,I_n)$ 表示输入常向量。系统初始条件为 $\boldsymbol{y}(\boldsymbol{t})=\boldsymbol{\phi}(\boldsymbol{t}),-\tau\leqslant t\leqslant 0$ 。

一般地,对激励函数作如下假设:

假设:激励函数 $g_j(\cdot)$ 连续、有界且

$$l_j^-\leqslant\frac{g_j(x)-g_j(y)}{x-y}\leqslant l_j^+,\forall x,y\in\mathbb{R},x\neq y,j=1,2,\cdots,n\text{。}\quad(2)$$

以上的假设能确保系统(1)存在平衡点

$\boldsymbol{y}^*=\left(y_1^*,y_2^*,\cdots,y_n^*\right)$。为以下讨论方便,作平移变换 $x_j(t)=y_j(t)-y_j^*$,则系统(1)变为:

$$\dot{\boldsymbol{x}}(t)=-\boldsymbol{C}\boldsymbol{x}(t)+\boldsymbol{A}\boldsymbol{f}\left(\boldsymbol{x}(t)\right)+\boldsymbol{B}\boldsymbol{f}\left(\boldsymbol{x}(t-\tau)\right)。\tag{3}$$

式(3)中:$\boldsymbol{x}(t)=\boldsymbol{\varphi}(t),\ -\tau\leqslant t\leqslant 0$;

$$\boldsymbol{f}\left(\boldsymbol{x}(t)\right)=\left(f_1\left(x_1(t)\right),f_2\left(x_2(t)\right),\cdots,f_n\left(x_n(t)\right)\right);$$

$$f_j\left(x_j(t)\right)=g_j\left(x_j(t)+y_j^*\right)-g_j\left(y_j^*\right)。\tag{4}$$

由假设及式(4),显然有:

$$l_j^-\leqslant\frac{f_j(s)}{s}\leqslant l_j^+,\ \forall s\neq 0, f_j(0)=0, j=1,2,\cdots,n。\tag{5}$$

则系统(3)相对于初始初始条件存在平衡点 $x(t)\equiv 0$。

引理2.1:(Jensen积分不等式)[14]对任意的对称正定阵 $\boldsymbol{M}=\boldsymbol{M}^{\mathrm{T}}>0$,标量 $\gamma>0$ 及使得如下积分有定义的向量值函数 $\boldsymbol{\omega}:[0,\gamma]\rightarrow\mathbb{R}^n$,下面的积分不等式成立:

$$\left(\int_0^\gamma\boldsymbol{\omega}(s)\mathrm{d}s\right)^{\mathrm{T}}\boldsymbol{M}\left(\int_0^\gamma\boldsymbol{\omega}(s)\mathrm{d}s\right)\leqslant\gamma\int_0^\gamma\boldsymbol{\omega}^{\mathrm{T}}(s)\boldsymbol{M}\boldsymbol{\omega}(s)\mathrm{d}s。$$

引理2.2:(Schur补引理)[29]对于给定的恰当维数的矩阵 $\boldsymbol{\Omega}_1$、$\boldsymbol{\Omega}_2$ 和 $\boldsymbol{\Omega}_3$,其中:$\boldsymbol{\Omega}_1^{\mathrm{T}}=\boldsymbol{\Omega}_1$,$\boldsymbol{\Omega}_2^{\mathrm{T}}=\boldsymbol{\Omega}_2>0$,则 $\boldsymbol{\Omega}_1+\boldsymbol{\Omega}_3^{\mathrm{T}}\boldsymbol{\Omega}_2^{-1}\boldsymbol{\Omega}_3<0$ 当且仅当

$$\begin{pmatrix}\boldsymbol{\Omega}_1 & \boldsymbol{\Omega}_3^{\mathrm{T}}\\ * & -\boldsymbol{\Omega}_2\end{pmatrix}<0\ 或\ \begin{pmatrix}-\boldsymbol{\Omega}_2 & \boldsymbol{\Omega}_3^{\mathrm{T}}\\ * & \boldsymbol{\Omega}_1\end{pmatrix}<0。$$

2 主要结果

定理3:对给定的 $\tau,m\geqslant 1$,若存在对称正定阵 $\boldsymbol{P}\in\mathbb{R}^{3n\times 3n}$,$\boldsymbol{Q}\in\mathbb{R}^{2mn\times 2mn}$,$\boldsymbol{R}_i\in\mathbb{R}^{n\times n}(i=1,2,3,4)$,$\boldsymbol{S}_i\in\mathbb{R}^{n\times n}(i=1,2,3)$,非负对角阵 $\boldsymbol{D}=\mathrm{diag}(d_1,d_2,\cdots,d_n)$,$\boldsymbol{E}=\mathrm{diag}(e_1,e_2,\cdots,e_n)$,$\boldsymbol{\Lambda}_i=\mathrm{diag}(\lambda_{i1},\lambda_{i2},\cdots,\lambda_{in}),i=1,2$,以及任意恰当维数的矩阵 $\boldsymbol{M}_i$、$\boldsymbol{N}_i$、$\boldsymbol{F}_i$、$\boldsymbol{G}_i(i=1,2)$,$\boldsymbol{H}_i$ $(i=1,2,3,4)$ 使得如下的LMI成立,则系统(3)的平衡点唯一且全局渐近稳定:

$$\boldsymbol{\Psi}=\begin{pmatrix}\boldsymbol{\Xi} & \sqrt{\frac{\tau}{m}}\tilde{\boldsymbol{M}} & \sqrt{\tau}\tilde{\boldsymbol{N}} & \frac{1}{\sqrt{2}}\frac{\tau}{m}\tilde{\boldsymbol{F}} & \frac{1}{\sqrt{2}}\tau\tilde{\boldsymbol{G}}\\ & -\boldsymbol{R}_1 & \boldsymbol{O} & \boldsymbol{O} & \boldsymbol{O}\\ * & & -\boldsymbol{R}_2 & \boldsymbol{O} & \boldsymbol{O}\\ & * & & -\boldsymbol{S}_1 & \boldsymbol{O}\\ & & * & & -\boldsymbol{S}_2\end{pmatrix}<\boldsymbol{O}。\tag{6}$$

式(6)中:

$$\boldsymbol{\Xi}=\mathrm{sym}\left(\boldsymbol{W}_{P_1}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{W}_{P_2}\right)+\mathrm{sym}\left(\boldsymbol{W}_D^{\mathrm{T}}\tilde{\boldsymbol{D}}\boldsymbol{W}_E\right)+\boldsymbol{W}_{\tilde{Q}}^{\mathrm{T}}\tilde{\boldsymbol{Q}}\boldsymbol{W}_{\tilde{Q}}+$$
$$\boldsymbol{W}_{\tilde{R}}^{\mathrm{T}}\tilde{\boldsymbol{R}}\boldsymbol{W}_{\tilde{R}}+\boldsymbol{W}_{\tilde{S}}^{\mathrm{T}}\tilde{\boldsymbol{S}}\boldsymbol{W}_{\tilde{S}}+\mathrm{sym}\left(\tilde{\boldsymbol{M}}\hat{\boldsymbol{M}}\right)+\mathrm{sym}\left(\tilde{\boldsymbol{N}}\hat{\boldsymbol{N}}\right)+$$
$$\mathrm{sym}\left(\tilde{\boldsymbol{F}}\hat{\boldsymbol{F}}\right)+\mathrm{sym}\left(\tilde{\boldsymbol{G}}\hat{\boldsymbol{G}}\right)+\mathrm{sym}\left(\tilde{\boldsymbol{H}}\hat{\boldsymbol{H}}\right)+\mathrm{sym}\left(\boldsymbol{W}_{\tilde{\Lambda}_1}^{\mathrm{T}}\tilde{\boldsymbol{\Lambda}}\boldsymbol{W}_{\tilde{\Lambda}_2}\right)。$$

其中:

$$\boldsymbol{W}_{P_1}=\begin{pmatrix}\boldsymbol{I}_{n\times n} & & \boldsymbol{O}_{n\times(2m+7)n}\\ \boldsymbol{O}_{n\times(2m+3)n} & \boldsymbol{I}_{n\times n} & \boldsymbol{O}_{n\times 4n}\\ \boldsymbol{O}_{n\times(2m+6)n} & \boldsymbol{I}_{n\times n} & \boldsymbol{O}_{n\times n}\end{pmatrix};\ \boldsymbol{W}_{P_2}=\begin{pmatrix}\boldsymbol{O}_{n\times(2m+2)n} & & \boldsymbol{I}_{n\times n} & & \boldsymbol{O}_{n\times 5n}\\ \boldsymbol{I}_{n\times n} & & -\boldsymbol{I}_{n\times n} & & \boldsymbol{O}_{n\times(2m+6)n}\\ \boldsymbol{O}_{n\times(m+1)n} & \boldsymbol{I}_{n\times n} & \boldsymbol{O}_{n\times(m-1)n} & -\boldsymbol{I}_{n\times n} & \boldsymbol{O}_{n\times 6n}\end{pmatrix};$$

$$\boldsymbol{W}_D=\begin{pmatrix}-L_1 & \boldsymbol{O}_{n\times mn} & \boldsymbol{I}_{n\times n} & \boldsymbol{O}_{n\times(m+6)n}\\ L_2 & \boldsymbol{O}_{n\times mn} & -\boldsymbol{I}_{n\times n} & \boldsymbol{O}_{n\times(m+6)n}\end{pmatrix};\ \tilde{\boldsymbol{D}}=\begin{pmatrix}\boldsymbol{D} & \\ & \boldsymbol{E}\end{pmatrix};\ \boldsymbol{W}_E=\begin{pmatrix}\boldsymbol{O}_{n\times(2m+2)n} & \boldsymbol{I}_{n\times n} & \boldsymbol{O}_{n\times 5n}\\ \boldsymbol{O}_{n\times(2m+2)n} & \boldsymbol{I}_{n\times n} & \boldsymbol{O}_{n\times 5n}\end{pmatrix};$$

$$\boldsymbol{W}_{\tilde{Q}}^{\mathrm{T}}=\begin{pmatrix}\boldsymbol{I}_{mn\times mn} & \boldsymbol{O}_{mn\times mn} & \boldsymbol{O}_{n\times mn} & \boldsymbol{O}_{n\times mn}\\ \boldsymbol{O}_{n\times mn} & \boldsymbol{O}_{n\times mn} & \boldsymbol{I}_{mn\times mn} & \boldsymbol{O}_{mn\times mn}\\ & & \boldsymbol{O}_{n\times mn} & \boldsymbol{O}_{n\times mn}\\ \boldsymbol{O}_{mn\times mn} & \boldsymbol{I}_{mn\times mn} & \boldsymbol{O}_{mn\times mn} & \boldsymbol{I}_{mn\times mn}\\ \boldsymbol{O}_{7n\times mn} & \boldsymbol{O}_{7n\times mn} & \boldsymbol{O}_{6n\times mn} & \boldsymbol{O}_{6n\times mn}\end{pmatrix};\ \boldsymbol{W}_{\tilde{R}}^{\mathrm{T}}=\begin{pmatrix}\boldsymbol{O}_{(2m+2)n\times n} & \boldsymbol{O}_{(2m+2)n\times n} & \sqrt{\frac{\tau}{m}}\boldsymbol{I}_{n\times n} & \boldsymbol{O}_{(2m+3)n\times n} & \boldsymbol{O}_{(m+1)n\times n} & \boldsymbol{O}_{(2m+5)n\times n}\\ \sqrt{\frac{\tau}{m}}\boldsymbol{I}_{n\times n} & \sqrt{\tau}\boldsymbol{I}_{n\times n} & & \sqrt{\frac{m}{\tau}}\boldsymbol{I}_{n\times n} & \sqrt{\frac{\tau}{m}}\boldsymbol{I}_{n\times n} & \sqrt{\frac{m}{\tau}}\boldsymbol{I}_{n\times n}\\ \boldsymbol{O}_{5n\times n} & \boldsymbol{O}_{5n\times n} & \boldsymbol{O}_{(2m+7)n\times n} & \boldsymbol{O}_{4n\times n} & \boldsymbol{O}_{(m+6)n\times n} & \boldsymbol{O}_{2n\times n}\end{pmatrix};$$

$$\boldsymbol{W}_{\tilde{S}}^{\mathrm{T}}=\begin{pmatrix}\boldsymbol{O}_{(2m+2)n\times n} & \boldsymbol{O}_{(2m+2)n\times n} & \boldsymbol{O}_{(m+1)n\times n} & \boldsymbol{O}_{(2m+7)n\times n}\\ \frac{1}{\sqrt{2}}\frac{\tau}{m}\boldsymbol{I}_{n\times n} & \frac{\tau}{\sqrt{2}}\boldsymbol{I}_{n\times n} & \frac{1}{\sqrt{2}}\frac{\tau}{m}\boldsymbol{I}_{n\times n} & \\ \boldsymbol{O}_{5n\times n} & \boldsymbol{O}_{5n\times n} & \boldsymbol{O}_{(m+6)n\times n} & \sqrt{2}\frac{m}{\tau}\boldsymbol{I}_{n\times n}\end{pmatrix};\ \tilde{\boldsymbol{Q}}=\mathrm{diag}\left(Q,-Q\right);\ \tilde{\boldsymbol{R}}=\mathrm{diag}\left(R_1,R_2,R_3,-R_3,R_4,-R_4\right);$$

$$\tilde{\boldsymbol{S}}=\mathrm{diag}\left(S_1,S_2,S_3,-S_3\right);\ \tilde{\boldsymbol{\Lambda}}=\mathrm{diag}\left(\Lambda_1,\Lambda_2\right);\ \boldsymbol{L}_1=\mathrm{diag}\left(l_1^-,l_2^-,\cdots,l_n^-\right);\ \boldsymbol{L}_2=\mathrm{diag}\left(l_1^+,l_2^+,\cdots,l_n^+\right);$$

$$\tilde{\boldsymbol{M}}^{\mathrm{T}}=\left(\boldsymbol{M}_1^{\mathrm{T}},\boldsymbol{M}_2^{\mathrm{T}},\boldsymbol{O}_{n\times(2m+6)n}\right),\ \hat{\boldsymbol{M}}=\left(\boldsymbol{I}_{n\times n},-\boldsymbol{I}_{n\times n},\boldsymbol{O}_{n\times(2m+6)n}\right);$$

$$\tilde{\boldsymbol{N}}^{\mathrm{T}}=\left(\boldsymbol{N}_1^{\mathrm{T}},\boldsymbol{O}_{n\times(m-1)n},\boldsymbol{N}_2^{\mathrm{T}},\boldsymbol{O}_{n\times(m+7)n}\right);\ \hat{\boldsymbol{N}}=\left(\boldsymbol{I}_{n\times n},\boldsymbol{O}_{n\times(m-1)n},-\boldsymbol{I}_{n\times n},\boldsymbol{O}_{n\times(m+7)n}\right);$$

$$\tilde{\boldsymbol{F}}^{\mathrm{T}}=\left(\boldsymbol{F}_1^{\mathrm{T}},\boldsymbol{O}_{n\times(2m+2)n},\boldsymbol{F}_2^{\mathrm{T}},\boldsymbol{O}_{n\times 4n}\right);\ \hat{\boldsymbol{F}}=\left(\frac{\tau}{m}\boldsymbol{I}_{n\times n},\boldsymbol{O}_{n\times(2m+2)n},-\boldsymbol{I}_{n\times n},\boldsymbol{O}_{n\times 4n}\right);$$

$$\tilde{G}^{\mathrm{T}}=\left(G_1^{\mathrm{T}},O_{n\times(2m+3)n},G_2^{\mathrm{T}},O_{n\times 3n}\right);\ \hat{G}=\left(\tau I_{n\times n},O_{n\times(2m+3)n},-I_{n\times n},O_{n\times 3n}\right);$$

$$\tilde{H}^{\mathrm{T}}=\left(H_1^{\mathrm{T}},O_{n\times mn},H_3^{\mathrm{T}},O_{n\times(m-1)n},H_4^{\mathrm{T}},H_2^{\mathrm{T}},O_{n\times 5n}\right);\ \hat{H}=\left(-C,O_{n\times mn},A,O_{n\times(m-1)n},B,-I_{n\times n},O_{n\times 5n}\right);$$

$$W_{\bar{\Lambda}_1}=\begin{pmatrix}-L_1 & O_{n\times mn} & & I_{n\times n} & O_{n\times(m+6)n}\\ O_{n\times mn} & -L_1 & O_{n\times mn} & I_{n\times n} & O_{n\times 6n}\end{pmatrix};\ W_{\bar{\Lambda}_2}=\begin{pmatrix}L_2 & O_{n\times mn} & & -I_{n\times n} & O_{n\times(m+6)n}\\ O_{n\times mn} & L_2 & O_{n\times mn} & -I_{n\times n} & O_{n\times 6n}\end{pmatrix}。$$

证明: 由Brouwer不动点定理不难证明系统平衡点的唯一性,此处略。只证平衡点全局渐近稳定。为此,构造如下的增广Lyapunov-Krasovskii泛函 $V(t)=V_1(t)+V_2(t)+V_3(t)+V_4(t)$。其中:

$$V_1(t)=\varsigma^{\mathrm{T}}(t)P\varsigma(t)+2\sum_{j=1}^{n}\left(d_j\int_0^{x_j(t)}\left(f_j(s)-l_j^- s\right)\mathrm{d}s+e_j\int_0^{x_j(t)}\left(l_j^+ s-f_j(s)\right)\mathrm{d}s\right);\ V_2(t)=\int_{t-\frac{\tau}{m}}^{t}\eta^{\mathrm{T}}(s)Q\eta(s)\mathrm{d}s;$$

$$V_3(t)=\int_{-\frac{\tau}{m}}^{0}\int_{t+\theta}^{t}\dot{x}^{\mathrm{T}}(s)R_1\dot{x}(s)\mathrm{d}s\mathrm{d}\theta+\int_{-\tau}^{0}\int_{t+\theta}^{t}\dot{x}^{\mathrm{T}}(s)R_2\dot{x}(s)\mathrm{d}s\mathrm{d}\theta+\int_{-\frac{\tau}{m}}^{0}\int_{t+\theta}^{t}x^{\mathrm{T}}(s)R_3x(s)\mathrm{d}s\mathrm{d}\theta+\int_{-\frac{\tau}{m}}^{0}\int_{t+\theta}^{t}f^{\mathrm{T}}(x(s))R_4f(x(s))\mathrm{d}s\mathrm{d}\theta;$$

$$V_4(t)=\int_{-\frac{\tau}{m}}^{0}\int_{\theta}^{0}\int_{t+\lambda}^{t}\dot{x}^{\mathrm{T}}(s)S_1\dot{x}(s)\mathrm{d}s\mathrm{d}\lambda\mathrm{d}\theta+\int_{-\tau}^{0}\int_{\theta}^{0}\int_{t+\lambda}^{t}\dot{x}^{\mathrm{T}}(s)S_2\dot{x}(s)\mathrm{d}s\mathrm{d}\lambda\mathrm{d}\theta+\int_{-\frac{\tau}{m}}^{0}\int_{\theta}^{0}\int_{t+\lambda}^{t}f^{\mathrm{T}}(x(s))S_3f(x(s))\mathrm{d}s\mathrm{d}\lambda\mathrm{d}\theta;$$

$$\varsigma^{\mathrm{T}}(t)=\left(x^{\mathrm{T}}(t),\int_{t-\frac{\tau}{m}}^{t}x^{\mathrm{T}}(s)\mathrm{d}s,\int_{t-\tau}^{t}f^{\mathrm{T}}(x(s))\mathrm{d}s\right);$$

$$\eta^{\mathrm{T}}(t)=\left(x^{\mathrm{T}}(t),x^{\mathrm{T}}\left(t-\frac{\tau}{m}\right),\cdots,x^{\mathrm{T}}\left(t-\frac{m-1}{m}\tau\right),f^{\mathrm{T}}(x(t)),f^{\mathrm{T}}\left(x\left(t-\frac{\tau}{m}\right)\right),\cdots,f^{\mathrm{T}}\left(x\left(t-\frac{m-1}{m}\tau\right)\right)\right)。$$

现在计算 $V(t)$ 沿系统(3)对时间 t 的导数:

$$\dot{V}_1(t)=2\varsigma^{\mathrm{T}}(t)P\dot{\varsigma}(t)+2\left(f^{\mathrm{T}}(x(t))-x^{\mathrm{T}}(t)L_1\right)D\dot{x}(t)+2\left(x^{\mathrm{T}}(t)L_2-f^{\mathrm{T}}(x(t))\right)E\dot{x}(t);$$

$$\dot{V}_2(t)=\eta^{\mathrm{T}}(t)Q\eta(t)-\eta^{\mathrm{T}}\left(t-\frac{\tau}{m}\right)Q\eta\left(t-\frac{\tau}{m}\right);$$

$$\begin{aligned}\dot{V}_3(t)=&\frac{\tau}{m}\dot{x}^{\mathrm{T}}(t)R_1\dot{x}(t)+\tau\dot{x}^{\mathrm{T}}(t)R_2\dot{x}(t)+\frac{\tau}{m}x^{\mathrm{T}}(t)R_3x(t)+\frac{\tau}{m}f^{\mathrm{T}}(x(t))R_4f^{\mathrm{T}}(x(t))-\int_{t-\frac{\tau}{m}}^{t}\dot{x}^{\mathrm{T}}(s)R_1\dot{x}(s)\mathrm{d}s-\\&\int_{t-\tau}^{t}\dot{x}^{\mathrm{T}}(s)R_2\dot{x}(s)\mathrm{d}s-\int_{t-\frac{\tau}{m}}^{t}x^{\mathrm{T}}(s)R_3x(s)\mathrm{d}s-\int_{t-\frac{\tau}{m}}^{t}f^{\mathrm{T}}(x(s))R_4f(x(s))\mathrm{d}s;\end{aligned}\tag{7}$$

$$\begin{aligned}\dot{V}_4(t)=&\frac{1}{2}\left(\frac{\tau}{m}\right)^2\dot{x}^{\mathrm{T}}(t)S_1\dot{x}(t)+\frac{1}{2}\tau^2\dot{x}^{\mathrm{T}}(t)S_2\dot{x}(t)+\frac{1}{2}\left(\frac{\tau}{m}\right)^2f^{\mathrm{T}}(x(t))S_3f(x(t))-\\&\int_{-\frac{\tau}{m}}^{0}\int_{t+\theta}^{t}\dot{x}^{\mathrm{T}}(s)S_1\dot{x}(s)\mathrm{d}s\mathrm{d}\theta-\int_{-\tau}^{0}\int_{t+\theta}^{t}\dot{x}^{\mathrm{T}}(s)S_2\dot{x}(s)\mathrm{d}s\mathrm{d}\theta-\int_{-\frac{\tau}{m}}^{0}\int_{t+\theta}^{t}f^{\mathrm{T}}(x(s))S_3f(x(s))\mathrm{d}s\mathrm{d}\theta。\end{aligned}\tag{8}$$

对 $\dot{V}_3(t)$ 中的最后两项和 $\dot{V}_4(t)$ 的最后一项,由引理2.1,有:

$$-\int_{t-\frac{\tau}{m}}^{t}x^{\mathrm{T}}(s)R_3x(s)\mathrm{d}s\leqslant-\frac{m}{\tau}\left(\int_{t-\frac{\tau}{m}}^{t}x(s)\mathrm{d}s\right)^{\mathrm{T}}R_3\left(\int_{t-\frac{\tau}{m}}^{t}x(s)\mathrm{d}s\right);\tag{9}$$

$$-\int_{t-\frac{\tau}{m}}^{t}f^{\mathrm{T}}(x(s))R_4f(x(s))\mathrm{d}s\leqslant-\frac{m}{\tau}\left(\int_{t-\frac{\tau}{m}}^{t}f(x(s))\mathrm{d}s\right)^{\mathrm{T}}R_4\left(\int_{t-\frac{\tau}{m}}^{t}f(x(s))\mathrm{d}s\right);\tag{10}$$

$$-\int_{-\frac{\tau}{m}}^{0}\int_{t+\theta}^{t}f^{\mathrm{T}}(x(s))S_3f(x(s))\mathrm{d}s\mathrm{d}\theta\leqslant-2\left(\frac{m}{\tau}\right)^2\left(\int_{-\frac{\tau}{m}}^{0}\int_{t+\theta}^{t}f(x(s))\mathrm{d}s\mathrm{d}\theta\right)^{\mathrm{T}}S_3\left(\int_{-\frac{\tau}{m}}^{0}\int_{t+\theta}^{t}f(x(s))\mathrm{d}s\mathrm{d}\theta\right)。\tag{11}$$

由Newton-Leibniz公式,存在恰当维数的矩阵 $M_i,N_i,F_i,G_i(i=1,2)$, $H_i\ (i=1,2,3,4)$ 使得下面的等式成立:

$$2\left(x^{\mathrm{T}}(t)M_1+x^{\mathrm{T}}\left(t-\frac{\tau}{m}\right)M_2\right)\left(x(t)-x\left(t-\frac{\tau}{m}\right)-\int_{t-\frac{\tau}{m}}^{t}\dot{x}(s)\mathrm{d}s\right)=0;\tag{12}$$

$$2\left(x^{\mathrm{T}}(t)N_1+x^{\mathrm{T}}(t-\tau)N_2\right)\left(x(t)-x(t-\tau)-\int_{t-\tau}^{t}\dot{x}(s)\mathrm{d}s\right)=0;\tag{13}$$

$$2\left(\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{F}_1+\int_{t-\frac{\tau}{m}}^{t}\boldsymbol{x}^{\mathrm{T}}(s)\mathrm{d}s\boldsymbol{F}_2\right)\left(\frac{\tau}{m}\boldsymbol{x}(t)-\int_{t-\frac{\tau}{m}}^{t}\boldsymbol{x}(s)\mathrm{d}s-\int_{-\frac{\tau}{m}}^{0}\int_{t+\theta}^{t}\dot{\boldsymbol{x}}(s)\mathrm{d}s\mathrm{d}\theta\right)=0\ ; \tag{14}$$

$$2\left(\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{G}_1+\int_{t-\tau}^{t}\boldsymbol{x}^{\mathrm{T}}(s)\mathrm{d}s\boldsymbol{G}_2\right)\left(\tau\boldsymbol{x}(t)-\int_{t-\tau}^{t}\boldsymbol{x}(s)\mathrm{d}s-\int_{-\tau}^{0}\int_{t+\theta}^{t}\dot{\boldsymbol{x}}(s)\mathrm{d}s\mathrm{d}\theta\right)=0\ ; \tag{15}$$

$$2\left(\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{H}_1+\dot{\boldsymbol{x}}^{\mathrm{T}}(t)\boldsymbol{H}_2+\boldsymbol{f}^{\mathrm{T}}\left(\boldsymbol{x}(t)\right)\boldsymbol{H}_3+\boldsymbol{f}^{\mathrm{T}}\left(\boldsymbol{x}(t-\tau)\right)\boldsymbol{H}_4\right)\cdot\left(-\boldsymbol{C}\boldsymbol{x}(t)+\boldsymbol{A}\boldsymbol{f}\left(\boldsymbol{x}(t)\right)+\boldsymbol{B}\boldsymbol{f}\left(\boldsymbol{x}(t-\tau)\right)-\dot{\boldsymbol{x}}(t)\right)=0\ 。 \tag{16}$$

进一步,还有:

$$-2\boldsymbol{\xi}^{\mathrm{T}}(t)\tilde{\boldsymbol{M}}\int_{t-\frac{\tau}{m}}^{t}\dot{\boldsymbol{x}}(s)\mathrm{d}s\leqslant\frac{\tau}{m}\boldsymbol{\xi}^{\mathrm{T}}(t)\tilde{\boldsymbol{M}}\boldsymbol{R}_1^{-1}\tilde{\boldsymbol{M}}^{\mathrm{T}}\boldsymbol{\xi}(t)+\int_{t-\frac{\tau}{m}}^{t}\dot{\boldsymbol{x}}^{\mathrm{T}}(s)\boldsymbol{R}_1\dot{\boldsymbol{x}}(s)\mathrm{d}s\ ; \tag{17}$$

$$-2\boldsymbol{\xi}^{\mathrm{T}}(t)\tilde{\boldsymbol{N}}\int_{t-\tau}^{t}\dot{\boldsymbol{x}}(s)\mathrm{d}s\leqslant\tau\boldsymbol{\xi}^{\mathrm{T}}(t)\tilde{\boldsymbol{N}}\boldsymbol{R}_2^{-1}\tilde{\boldsymbol{N}}^{\mathrm{T}}\boldsymbol{\xi}(t)+\int_{t-\tau}^{t}\dot{\boldsymbol{x}}^{\mathrm{T}}(s)\boldsymbol{R}_2\dot{\boldsymbol{x}}(s)\mathrm{d}s\ ; \tag{18}$$

$$-2\boldsymbol{\xi}^{\mathrm{T}}(t)\tilde{\boldsymbol{F}}\int_{-\frac{\tau}{m}}^{0}\int_{t+\theta}^{t}\dot{\boldsymbol{x}}(s)\mathrm{d}s\mathrm{d}\theta\leqslant\frac{1}{2}\left(\frac{\tau}{m}\right)^2\boldsymbol{\xi}^{\mathrm{T}}(t)\tilde{\boldsymbol{F}}\boldsymbol{S}_1^{-1}\tilde{\boldsymbol{F}}^{\mathrm{T}}\boldsymbol{\xi}(t)+\int_{-\frac{\tau}{m}}^{0}\int_{t+\theta}^{t}\dot{\boldsymbol{x}}(s)\boldsymbol{S}_1\dot{\boldsymbol{x}}(s)\mathrm{d}s\mathrm{d}\theta\ ; \tag{19}$$

$$-2\boldsymbol{\xi}^{\mathrm{T}}(t)\tilde{\boldsymbol{G}}\int_{-\tau}^{0}\int_{t+\theta}^{t}\dot{\boldsymbol{x}}(s)\mathrm{d}s\mathrm{d}\theta\leqslant\frac{1}{2}\tau^2\boldsymbol{\xi}^{\mathrm{T}}(t)\tilde{\boldsymbol{G}}\boldsymbol{S}_2^{-1}\tilde{\boldsymbol{G}}^{\mathrm{T}}\boldsymbol{\xi}(t)+\int_{-\tau}^{0}\int_{t+\theta}^{t}\dot{\boldsymbol{x}}(s)\boldsymbol{S}_2\dot{\boldsymbol{x}}(s)\mathrm{d}s\mathrm{d}\theta\ 。 \tag{20}$$

由式(5),对任意的非负标量 $\lambda_{ij}\geqslant 0,\ i=1,2,\ j=1,2,\cdots,n$,下面的不等式成立:

$$2\sum_{j=1}^{n}\lambda_{1j}\left(f_j\left(x_j(t)\right)-l_j^-x_j(t)\right)\left(l_j^+x_j(t)-f_j\left(x_j(t)\right)\right)\geqslant 0\ ;$$

$$2\sum_{j=1}^{n}\lambda_{1j}\left(f_j\left(x_j(t-\tau)\right)-l_j^-x_j(t-\tau)\right)\left(l_j^+x_j(t-\tau)-f_j\left(x_j(t-\tau)\right)\right)\geqslant 0\ 。$$

写成矩阵向量形式,也即:

$$2\left(\boldsymbol{f}^{\mathrm{T}}\left(\boldsymbol{x}(t)\right)-\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{L}_1\right)\boldsymbol{\varLambda}_1\left(\boldsymbol{L}_2\boldsymbol{x}(t)-\boldsymbol{f}\left(\boldsymbol{x}(t)\right)\right)\geqslant 0\ ; \tag{21}$$

$$2\left(\boldsymbol{f}^{\mathrm{T}}\left(\boldsymbol{x}(t-\tau)\right)-\boldsymbol{x}^{\mathrm{T}}(t-\tau)\boldsymbol{L}_1\right)\boldsymbol{\varLambda}_2\left(\boldsymbol{L}_2\boldsymbol{x}(t-\tau)-\boldsymbol{f}\left(\boldsymbol{x}(t-\tau)\right)\right)\geqslant 0\ 。 \tag{22}$$

由式(7)~(22),经计算整理得:

$$\dot{\boldsymbol{V}}(t)\leqslant\boldsymbol{\xi}^{\mathrm{T}}(t)\boldsymbol{\varXi}\boldsymbol{\xi}(t)+\frac{\tau}{m}\boldsymbol{\xi}^{\mathrm{T}}(t)\tilde{\boldsymbol{M}}\boldsymbol{R}_1^{-1}\tilde{\boldsymbol{M}}^{\mathrm{T}}\boldsymbol{\xi}(t)+\tau\boldsymbol{\xi}^{\mathrm{T}}(t)\tilde{\boldsymbol{N}}\boldsymbol{R}_2^{-1}\tilde{\boldsymbol{N}}^{\mathrm{T}}\boldsymbol{\xi}(t)+\frac{1}{2}\left(\frac{\tau}{m}\right)^2\boldsymbol{\xi}^{\mathrm{T}}(t)\tilde{\boldsymbol{F}}\boldsymbol{S}_1^{-1}\tilde{\boldsymbol{F}}^{\mathrm{T}}\boldsymbol{\xi}(t)+\frac{1}{2}\tau^2\boldsymbol{\xi}^{\mathrm{T}}(t)\tilde{\boldsymbol{G}}\boldsymbol{S}_2^{-1}\tilde{\boldsymbol{G}}^{\mathrm{T}}\boldsymbol{\xi}(t)\ 。$$

其中:

$$\boldsymbol{\xi}^{\mathrm{T}}(t)=\Bigg(\boldsymbol{\eta}^{\mathrm{T}}(t),\boldsymbol{x}^{\mathrm{T}}(t-\tau),\boldsymbol{f}^{\mathrm{T}}\left(\boldsymbol{x}(t)\right),\boldsymbol{f}^{\mathrm{T}}\left(\boldsymbol{x}\left(t-\frac{\tau}{m}\right)\right),\cdots,$$

$$\boldsymbol{f}^{\mathrm{T}}\left(\boldsymbol{x}(t-\tau)\right),\dot{\boldsymbol{x}}^{\mathrm{T}}(t),\int_{t-\frac{\tau}{m}}^{t}\boldsymbol{x}^{\mathrm{T}}(s)\mathrm{d}s,\int_{t-\tau}^{t}\boldsymbol{x}^{\mathrm{T}}(s)\mathrm{d}s,\int_{t-\frac{\tau}{m}}^{t}\boldsymbol{f}^{\mathrm{T}}\left(\boldsymbol{x}(s)\right)\mathrm{d}s,\int_{t-\tau}^{t}\boldsymbol{f}^{\mathrm{T}}\left(\boldsymbol{x}(s)\right)\mathrm{d}s,\int_{-\frac{\tau}{m}}^{0}\int_{t+\theta}^{t}\boldsymbol{f}^{\mathrm{T}}\left(\boldsymbol{x}(s)\right)\mathrm{d}s\mathrm{d}\theta\Bigg)$$

是 $(2m+8)n$ 维行向量,矩阵 $\boldsymbol{\varXi}$ 如前定义。

由式(6)及引理2.2知,存在正数 ε 使下式成立:

$$\boldsymbol{\varPsi}<\mathrm{diag}(-\varepsilon\boldsymbol{I},\boldsymbol{O},\boldsymbol{O},\cdots,\boldsymbol{O})\ 。$$

从而,有 $\dot{\boldsymbol{V}}(t)\leqslant\boldsymbol{\xi}^{\mathrm{T}}(t)\boldsymbol{\varPsi}\boldsymbol{\xi}(t)<-\varepsilon\left\|\boldsymbol{x}(t)\right\|^2$ 。

这意味着系统(3)的平衡点全局渐近稳定,证毕。

注1: 通过构造一个更具一般性的增Lyapunov-Krasovskii泛函,定理3给出了一个时滞相依全局渐近稳定新判据,值得指出的是,这里 $\boldsymbol{V}_1(t)$ 的第一项, $\boldsymbol{V}_2(t)$ 中的二重积分项, $\boldsymbol{V}_3(t)$ 中的三重积分项以及 $\boldsymbol{V}_4(t)$ 都有别于相关文献,它们对降低稳定性判据中的保守性起着重要作用。

注2: 时滞分割也是降低保守性的重要因素。随时滞分割数目的增加,保守性也随着降低。即便在分割数目 $m=1$,即不分割时,下面的数值实例将说明本文结果也优于相关文献结果。

3 数值实例

例:考虑4阶时滞神经网络系统[7-8],参数如下:

$\boldsymbol{C}=\mathrm{diag}(1.276\,9,\ 0.623\,1,\ 0.923\,0,\ 0.448\,0)$;

$$\boldsymbol{A}=\begin{pmatrix}-0.037\,3 & 0.485\,2 & -0.335\,1 & 0.233\,6\\ -1.603\,3 & 0.598\,8 & -0.322\,4 & 1.235\,2\\ 0.339\,4 & -0.086\,0 & -0.382\,4 & -0.578\,5\\ -0.131\,1 & 0.325\,3 & -0.953\,4 & -0.501\,5\end{pmatrix};$$

$$B=\begin{pmatrix} 0.8674 & -1.2405 & -0.5325 & 0.0220 \\ 0.0474 & -0.9164 & 0.0360 & 0.9816 \\ 1.8495 & 2.6117 & -0.3788 & 0.8428 \\ -2.0413 & 0.5179 & 1.1734 & 0.2775 \end{pmatrix};$$

$$L_1=\mathrm{diag}(0,0,0,0);$$

$$L_2=\mathrm{diag}(0.1137,\ 0.1279,\ 0.7994,\ 0.2368)。$$

根据文文献[7-8]中稳定性判据,可计算得其时滞最大允许上界(MAUB)分别为3.584和3.589 8。而用本文判据,当时滞分割数目 m 分别取1、2、3、4、5时,相应MAUB分别为3.784 9,4.074 3,4.192 7,4.375 3和4.375 5,这说明了本文结果的有效性且显然是优于文[7-8]的结果的。但是由于矩阵变量 $\boldsymbol{P}$、$\boldsymbol{Q}$(尤其是后者)的维数较大,这给计算带来较大负担,而且当 m 分别取3、4、5时,MAUB的改善并不明显。通过比较,为获得较满意的MAUB,这里取 $m=3$ 更合适一些。

4 结语

本文通过构造一个新的增广Lyapunov-Krasovskii泛函,利用时滞分割技术获得了一个改善的系统时滞相依全局渐近稳定性充分条件,该条件以LMI得形式给出易于通过标准数字软件包计算、检验。数值实例表明时滞分割技术能有效降低系统稳定性条件的保守性。

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Delay-Dependent Global Stability of Neural Networks With Time Delay Based on Delay Fractitioning Technique

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Abstract: In this paper, a delay-dependent stability sufficient condition was obtained by a newly constructed Lyapunov-Krasovskii functional together with delay fractioning technique, free weighing matrix method and Jensen integral inequality, which was in form of LMIs and was less conservative than the existing ones.

Keywords: time-delay neural networks (DNN); global asymptotically stability; delay fractioning technique; free weighting matrix; Jensen integral inequality