文章编号:1673-1522(2019)03-0328-09 DOI: 10.7682/j.issn.1673-1522.2019.03.012

一类具有时变和分布时滞的神经网络系统的时滞相依全局稳定性分析

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摘 要:通过构造一个新的增广Lyapunov-Krasovskii泛函,对时变时滞函数的下界和定常分布时滞同时使用时滞分割,获得了一类同时具有时变时滞和分布时滞的神经网络系统以LMI形式表达的时滞相依全局渐近稳定条件,并通过数值实例表明本文方法的有效性和稳定条件更低的保守性。

关键词:分布时滞;时滞分割;时滞相依;全局渐近稳定

中图分类号:O175.13 **文献标志码**:A

由于在诸如图像处理、模式识别、联想记忆以及优化问题等方面的应用和潜在应用,近年来,神经网络系统得到了广泛的研究[1-2]。众所周知,神经网络系统的应用极大地依赖于其动力学行为,尤其是其平衡点的存在性和稳定性。同时,由于放大器转换速度的有限和信息处理速度的有限,时滞现象在神经网络系统中往往是不可避免的,时滞也常使得系统不稳定或产生振荡。因此,时滞神经网络系统的稳定性研究在理论和实践中都具有重要意义,已有不少关于时滞神经网络系统时滞相依或时滞独立的稳定性研究结果[3-6]。一般说来,时滞相依稳定性结果中包含了时滞的信息,因而通常比时滞独立的稳定性结果具有更低的保守性,尤其是对于小时滞神经网络系统来说这种现象更为明显。也正因为如此,使系统达到全局稳定的时滞最大可能取值——时滞最大允许上界(MAUB)成为学者们极其关注的研究指标,MAUB同时也是系统稳定性条件具有更低保守性的标志性指标,这方面的研究成果也不断被提出。然而,人们已经认识到,传统的Lyapunov-Krasovskii泛函的构造对系统稳定性条件保守性的降低作用甚小。因此,增广Lyapunov-Krasovskii泛函的构造就成为降低系统稳定性条件保守性的必然选择之一[7]。

近年来,时滞分割逐渐成为降低保守性的有效手段之一,例如,具有定常时滞的神经网络系统[8],具有时变时滞的神经网络系统[9-13],以及具有多时滞成分的连续系统[14],离散系统[15]以和一类中立型时滞系统[16],都利用了时滞分割技术获得系统的全局稳定性条件。文献[17]基于时滞分割技术研究了一类具有分布时滞的线性分数阶不确定性系统的稳定性及鲁棒稳定性问题,文献[18]首次将这种方法扩展至同时具有离散和分布时滞的中立型系统的稳定性研究。尽管上述文献的稳定性结果已具有较低的保守性,但通过构造恰当的增广Lyapunov-Krasovskii泛函,并使用更好的定界技巧估计泛函导数的上界,可望获得比上述文献具有更低保守性的系统稳定性结果。

基于以上分析,本文首先构造一个新的增广Lyapunov-Krasovskii泛函;再利用时滞分割技术对时变时滞函数的下界、定常分布时滞进行不同分割,并借助自由权矩阵和Jensen积分不等式等手段对泛函导数的上界进行更精细的估计、定界,获得以LMIs形式描述的具有更低保守性的系统时滞相依全局渐近稳定性判定条件;最后,以数值实例说明了方法的有效性。本文中:$\mathbb{R}^{n\times m}$表示$n\times m$实矩阵空间;上标T表示转置;$\boldsymbol{X}\geqslant\boldsymbol{Y}$($\boldsymbol{X}>\boldsymbol{Y}$)表示$\boldsymbol{X}-\boldsymbol{Y}$半正定(正定),其中$\boldsymbol{X}$、$\boldsymbol{Y}$均为对称阵;$\boldsymbol{I}_{n\times n}$、$\boldsymbol{0}_{n\times n}$分别表示$n\times n$单位阵和零矩阵;记号$*$表示对称矩阵中相应的对称块;$\mathrm{sym}(\boldsymbol{A})=\boldsymbol{A}+\boldsymbol{A}^{\mathrm{T}}$;$r$表示分布时滞长。

1 问题描述

考虑如下同时具有时变和分布时滞的神经网络系统:

$$\begin{aligned}\dot{y}(t)&=-\boldsymbol{C}y(t)+\boldsymbol{A}g\big(y(t)\big)+\\&\boldsymbol{B}g\Big(y\big(t-\tau(t)\big)\Big)+\boldsymbol{D}\int_{t-r}^{t}g\big(y(s)\big)\mathrm{d}s+\boldsymbol{I},\\&y(t)=\boldsymbol{\psi}(t),-\tau\leqslant t\leqslant 0。\end{aligned}\tag{1}$$

收稿日期:2019-04-24; **修回日期**:2019-05-28

基金项目:国家自然科学基金资助项目(11802338)

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式(1)中:n 为系统中神经元数目,$\boldsymbol{y}(t)=\left(y_1(t),y_2(t),\cdots,y_n(t)\right)^{\mathrm{T}}$ 为神经元状态向量;$\boldsymbol{g}\left(\boldsymbol{y}(t)\right)=\left(g_1\left(y_1(t)\right),g_2\left(y_2(t)\right),\cdots,g_n\left(y_n(t)\right)\right)^{\mathrm{T}}$ 为神经元激励函数;$\boldsymbol{C}=\mathrm{diag}(c_1,c_2,\cdots,c_n)$ 为神经元自反馈系数矩阵;$\boldsymbol{A},\boldsymbol{B},\boldsymbol{D}\in\mathbb{R}^{n\times n}$ 分别表示神经元连接、时滞连接和分布连接矩阵;$\boldsymbol{I}=\mathrm{diag}(I_1,I_2,\cdots,I_n)$ 为系统输入常向量;$\boldsymbol{y}(t)=\boldsymbol{\psi}(t),-\tau\leqslant t\leqslant 0$ 为初始条件。

一般,对激励函数和时变时滞函数作如下假设:

H1:激励函数 $g_j(*)$ 连续、有界,且 $l_j^-\leqslant\dfrac{g_j(x)-g_j(y)}{x-y}\leqslant l_j^+$,$\forall x,y\in\mathbb{R}$,$x\neq y$,$j=1,2,\cdots,n$。

H2:时变时滞函数 $\tau(t)$ 可微,且 $0<\tau_1\leqslant\tau(t)\leqslant\tau_2$,$\dot{\tau}(t)\leqslant\tau_3<1$,其中 τ_1、τ_2、τ_3 均为常数。

对于给定的初始条件,上面的假设H1保证了系统(1)平衡点 $\boldsymbol{y}^*=\left(y_1^*,y_2^*,\cdots,y_n^*\right)$ 的存在性,作平移变换 $x_j(t)=y_j(t)-y_j^*$,则系统(1)变为:

$$\dot{\boldsymbol{x}}(t)=-\boldsymbol{C}\boldsymbol{x}(t)+\boldsymbol{A}\boldsymbol{f}\left(\boldsymbol{x}(t)\right)+\boldsymbol{B}\boldsymbol{f}\left(\boldsymbol{x}\left(t-\tau(t)\right)\right)+\boldsymbol{D}\int_{t-r}^{t}\boldsymbol{f}\left(\boldsymbol{x}(s)\right)\mathrm{d}s,$$
$$\boldsymbol{x}(t)=\boldsymbol{\varphi}(t),-\tau\leqslant t\leqslant 0。\tag{2}$$

式中,$f_j\left(x_j(t)\right)=g_j\left(x_j(t)+y_j^*\right)-g_j\left(y_j^*\right)$。

由假设H1,显然有 $l_j^-\leqslant\dfrac{f_j(s)}{s}\leqslant l_j^+$,$\forall s\neq 0$,$f_j(0)=0$,$j=1,2,\cdots,n$,则由Brouwer不动点定理可以证明在初始条件 $\boldsymbol{x}(t)=\boldsymbol{\varphi}(t),-\tau\leqslant t\leqslant 0$ 下,系统(2)存在平衡点 $\boldsymbol{x}(t)\equiv 0$。

引理1(Jensen积分不等式)[9]:对任意的对称正定阵 $\boldsymbol{M}=\boldsymbol{M}^{\mathrm{T}}>\boldsymbol{0}$,标量 $\tau_2>\tau_1>0$ 及使得如下积分有定义的向量值函数 $\boldsymbol{\omega}:[0,\gamma]\to\mathbb{R}^n$,下面的积分不等式成立:

1)$$-\int_{t-\tau_2}^{t-\tau_1}\boldsymbol{\omega}^{\mathrm{T}}(s)\boldsymbol{M}\boldsymbol{\omega}(s)\mathrm{d}s\leqslant-\frac{1}{\tau_2-\tau_1}\left(\int_{t-\tau_2}^{t-\tau_1}\boldsymbol{\omega}(s)\mathrm{d}s\right)^{\mathrm{T}}\boldsymbol{M}\left(\int_{t-\tau_2}^{t-\tau_1}\boldsymbol{\omega}(s)\mathrm{d}s\right);$$

2)$$-\int_{-\tau_2}^{-\tau_1}\int_{t+\theta}^{t}\boldsymbol{\omega}^{\mathrm{T}}(s)\boldsymbol{M}\boldsymbol{\omega}(s)\mathrm{d}s\mathrm{d}\theta\leqslant-\frac{2}{\tau_2^2-\tau_1^2}\left(\int_{-\tau_2}^{-\tau_1}\int_{t+\theta}^{t}\boldsymbol{\omega}(s)\mathrm{d}s\mathrm{d}\theta\right)^{\mathrm{T}}\boldsymbol{M}\left(\int_{-\tau_2}^{-\tau_1}\int_{t+\theta}^{t}\boldsymbol{\omega}(s)\mathrm{d}s\mathrm{d}\theta\right)。$$

引理2(Schur补引理)[19]:对于给定的对称矩阵 $\boldsymbol{S}=\begin{pmatrix}\boldsymbol{S}_{11}&\boldsymbol{S}_{12}\\\boldsymbol{S}_{12}^{\mathrm{T}}&\boldsymbol{S}_{22}\end{pmatrix}<0$,与下面2个条件等价:

1)$\boldsymbol{S}_{11}<0$,$\boldsymbol{S}_{22}-\boldsymbol{S}_{12}^{\mathrm{T}}\boldsymbol{S}_{11}^{-1}\boldsymbol{S}_{12}<0$;

2)$\boldsymbol{S}_{22}<0$,$\boldsymbol{S}_{11}-\boldsymbol{S}_{12}\boldsymbol{S}_{22}^{-1}\boldsymbol{S}_{12}^{\mathrm{T}}<0$。

2 主要结果

本节将在一个新构造的增广Lyapunov-Krasovskii泛函的基础上,对时变时滞函数的下界和定常分布时滞同时进行时滞分割,以推导系统时滞相依全局稳定性条件。首先引入如下的记号:

$\boldsymbol{L}_1=\mathrm{diag}\left(l_1^-,l_2^-,\cdots,l_n^-\right)$,

$\boldsymbol{L}_2=\mathrm{diag}\left(l_1^+,l_2^+,\cdots,l_n^+\right)$,

$\boldsymbol{L}_3=\mathrm{diag}\left(l_1^-l_1^+,l_2^-l_2^+,\cdots,l_n^-l_n^+\right)$,

$\boldsymbol{L}_4=\mathrm{diag}\left(\dfrac{l_1^-+l_1^+}{2},\dfrac{l_2^-+l_2^+}{2},\cdots,\dfrac{l_n^-+l_n^+}{2}\right)$,

$$\boldsymbol{\varsigma}^{\mathrm{T}}(t)=\left(\boldsymbol{x}^{\mathrm{T}}(t),\int_{t-\frac{\tau_1}{m}}^{t}\boldsymbol{x}^{\mathrm{T}}(s)\mathrm{d}s,\int_{-\frac{r}{l}}^{0}\int_{t+\theta}^{t}\boldsymbol{f}^{\mathrm{T}}\left(\boldsymbol{x}(s)\right)\mathrm{d}s\mathrm{d}\theta\right),$$

$$\boldsymbol{\eta}_1^{\mathrm{T}}(t)=\left(\boldsymbol{x}^{\mathrm{T}}(t),\boldsymbol{x}^{\mathrm{T}}\left(t-\frac{\tau_1}{m}\right),\cdots,\boldsymbol{x}^{\mathrm{T}}\left(t-\frac{m-1}{m}\tau_1\right)\right),$$

$$\boldsymbol{\eta}_2^{\mathrm{T}}(t)=\left(\int_{t-\frac{r}{l}}^{t}\boldsymbol{f}^{\mathrm{T}}\left(\boldsymbol{x}(s)\right)\mathrm{d}s,\int_{t-\frac{2r}{l}}^{t-\frac{r}{l}}\boldsymbol{f}^{\mathrm{T}}\left(\boldsymbol{x}(s)\right)\mathrm{d}s,\cdots,\int_{t-r}^{t-\frac{l-1}{l}r}\boldsymbol{f}^{\mathrm{T}}\left(\boldsymbol{x}(s)\right)\mathrm{d}s\right),$$

$$\boldsymbol{\eta}_3^{\mathrm{T}}(t)=\left(\boldsymbol{x}^{\mathrm{T}}(t),\boldsymbol{f}^{\mathrm{T}}\left(\boldsymbol{x}(t)\right)\right)。$$

定理1:对给定的 τ_1、τ_2、τ_3,正整数 $m,l\geqslant 1$,若存在对称正定矩阵 $\boldsymbol{P}\in\mathbb{R}^{3n\times 3n}$,$\boldsymbol{Q}_1\in\mathbb{R}^{mn\times mn}$,$\boldsymbol{Q}_2\in\mathbb{R}^{ln\times ln}$,$\boldsymbol{Q}_3\in\mathbb{R}^{2n\times 2n}$,$\boldsymbol{Z},\boldsymbol{R}_i\in\mathbb{R}^{n\times n}\ (i=1,2,3,4)$,$\boldsymbol{S}_i\in\mathbb{R}^{n\times n}\ (i=1,2)$,非负对角阵 $\boldsymbol{W}=\mathrm{diag}(w_1,w_2,\cdots,w_n)$,$\boldsymbol{\Delta}=\mathrm{diag}(\delta_1,\delta_2,\cdots,\delta_n)$,$\boldsymbol{\Lambda}_i=\mathrm{diag}(\lambda_{i1},\lambda_{i2},\cdots,\lambda_{in})$,$i=1,2$,以及任意恰当维数的矩阵 $\boldsymbol{M}_i$、$\boldsymbol{N}_i$、$\boldsymbol{F}_i$、$\boldsymbol{G}_j$($i=1,2;j=1,2,3$),使如下LMIs(3)成立,则系统(2)的平衡点是全局渐近稳定的,

$$\begin{pmatrix}\boldsymbol{\Xi}&\sqrt{\dfrac{\tau_1}{m}}\tilde{\boldsymbol{M}}&\sqrt{\tau_2-\tau_1}\tilde{\boldsymbol{N}}&\dfrac{1}{\sqrt{2}}\dfrac{\tau_1}{m}\tilde{\boldsymbol{F}}\\ *&-\boldsymbol{R}_1&\boldsymbol{0}&\boldsymbol{0}\\ *&*&-\boldsymbol{R}_3&\boldsymbol{0}\\ *&*&*&-\boldsymbol{S}_1\end{pmatrix}<0,\tag{3}$$

式中,$\boldsymbol{\Xi}$ 定义为:

$$\begin{aligned}\boldsymbol{\Xi}=&\,\mathrm{sym}\left(\boldsymbol{W}_{P_1}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{W}_{P_2}\right)+\mathrm{sym}\left(\boldsymbol{W}_w^{\mathrm{T}}\tilde{\boldsymbol{W}}\boldsymbol{W}_E\right)+\boldsymbol{W}_{Q_1}^{\mathrm{T}}\boldsymbol{Q}_1\boldsymbol{W}_{Q_1}-\boldsymbol{W}_{Q'_1}^{\mathrm{T}}\boldsymbol{Q}_1\boldsymbol{W}_{Q'_1}+\\&\boldsymbol{W}_{Q_2}^{\mathrm{T}}\boldsymbol{Q}_2\boldsymbol{W}_{Q_2}-\boldsymbol{W}_{Q'_2}^{\mathrm{T}}\boldsymbol{Q}_2\boldsymbol{W}_{Q'_2}+\boldsymbol{W}_{\tilde{Q}_3}^{\mathrm{T}}\tilde{\boldsymbol{Q}}_3\boldsymbol{W}_{\tilde{Q}_3}+\boldsymbol{W}_{\tilde{Z}}^{\mathrm{T}}\tilde{\boldsymbol{Z}}\boldsymbol{W}_{\tilde{Z}}+\boldsymbol{W}_{\tilde{R}}^{\mathrm{T}}\tilde{\boldsymbol{R}}\boldsymbol{W}_{\tilde{R}}+\boldsymbol{W}_{\tilde{S}}^{\mathrm{T}}\tilde{\boldsymbol{S}}\boldsymbol{W}_{\tilde{S}}+\\&\mathrm{sym}\left(\tilde{\boldsymbol{M}}\hat{\boldsymbol{M}}\right)+\mathrm{sym}\left(\tilde{\boldsymbol{N}}\hat{\boldsymbol{N}}\right)+\mathrm{sym}\left(\tilde{\boldsymbol{F}}\hat{\boldsymbol{F}}\right)+\mathrm{sym}\left(\tilde{\boldsymbol{G}}\hat{\boldsymbol{G}}\right)+\boldsymbol{W}_{\tilde{\Lambda}_1}^{\mathrm{T}}\tilde{\boldsymbol{\Lambda}}_1\boldsymbol{W}_{\tilde{\Lambda}_1}+\boldsymbol{W}_{\tilde{\Lambda}_2}^{\mathrm{T}}\tilde{\boldsymbol{\Lambda}}_2\boldsymbol{W}_{\tilde{\Lambda}_2}。\end{aligned}\tag{4}$$

式(4)中:

$$W_{P_1}=\begin{pmatrix} I_{n\times n} & 0_{n\times(m+l+7)n} & 0_{n\times n} & 0_{n\times n} \\ 0_{n\times n} & 0_{n\times(m+l+7)n} & I_{n\times n} & 0_{n\times n} \\ 0_{n\times n} & 0_{n\times(m+l+7)n} & 0_{n\times n} & I_{n\times n} \end{pmatrix};\ W_{P_2}=\begin{pmatrix} 0_{n\times n} & 0_{n\times n} & 0_{n\times(m+1)n} & I_{n\times n} & 0_{n\times n} & 0_{n\times 2n} & 0_{n\times n} & 0_{n\times(l+2)n} \\ I_{n\times n} & -I_{n\times n} & 0_{n\times(m+1)n} & 0_{n\times n} & 0_{n\times n} & 0_{n\times 2n} & 0_{n\times n} & 0_{n\times(l+2)n} \\ 0_{n\times n} & 0_{n\times n} & 0_{n\times(m+1)n} & 0_{n\times n} & \frac{r}{l}I_{n\times n} & 0_{n\times 2n} & -I_{n\times n} & 0_{n\times(l+2)n} \end{pmatrix};$$

$$W_W=\begin{pmatrix} -L_1 & 0_{n\times(m+3)n} & I_{n\times n} & 0_{n\times(l+5)n} \\ L_2 & 0_{n\times(m+3)n} & -I_{n\times n} & 0_{n\times(l+5)n} \end{pmatrix};\ \tilde{W}=\begin{pmatrix} W & \\ & \Delta \end{pmatrix},\ W_E=\begin{pmatrix} 0_{n\times(m+3)n} & I_{n\times n} & 0_{n\times(l+6)n} \\ 0_{n\times(m+3)n} & I_{n\times n} & 0_{n\times(l+6)n} \end{pmatrix};$$

$$W_{\tilde{Z}}=\begin{pmatrix} 0_{n\times mn} & I_{n\times n} & 0_{n\times n} & 0_{n\times n} & 0_{n\times(l+7)n} \\ 0_{n\times mn} & 0_{n\times n} & 0_{n\times n} & I_{n\times n} & 0_{n\times(l+7)n} \end{pmatrix};\ W_{Q_1}=\begin{pmatrix} I_{mn\times mn} & 0_{mn\times(l+10)n} \end{pmatrix};\ W_{Q'_1}=\begin{pmatrix} 0_{mn\times n} & I_{mn\times mn} & 0_{mn\times(l+9)n} \end{pmatrix};$$

$$W_{Q_2}=\begin{pmatrix} 0_{ln\times(m+7)n} & I_{ln\times ln} & 0_{ln\times 3n} \end{pmatrix};\ W_{Q'_2}=\begin{pmatrix} 0_{ln\times(m+8)n} & I_{ln\times ln} & 0_{ln\times 2n} \end{pmatrix};$$

$$W_{\tilde{Q}_3}=\begin{pmatrix} 0_{n\times mn} & I_{n\times n} & 0_{n\times n} & 0_{n\times 3n} & 0_{n\times n} & 0_{n\times n} & 0_{n\times(l+3)n} \\ 0_{n\times mn} & 0_{n\times n} & 0_{n\times n} & 0_{n\times 3n} & I_{n\times n} & 0_{n\times n} & 0_{n\times(l+3)n} \\ 0_{n\times mn} & 0_{n\times n} & I_{n\times n} & 0_{n\times 3n} & 0_{n\times n} & 0_{n\times n} & 0_{n\times(l+3)n} \\ 0_{n\times mn} & 0_{n\times n} & 0_{n\times n} & 0_{n\times 3n} & 0_{n\times n} & I_{n\times n} & 0_{n\times(l+3)n} \end{pmatrix};$$

$$W_{\tilde{\Lambda}_1}=\begin{pmatrix} I_{n\times n} & 0_{n\times(m+3)n} & 0_{n\times n} & 0_{n\times(l+5)n} \\ 0_{n\times n} & 0_{n\times(m+3)n} & I_{n\times n} & 0_{n\times(l+5)n} \end{pmatrix};\ W_{\tilde{\Lambda}_2}=\begin{pmatrix} 0_{n\times mn} & I_{n\times n} & 0_{n\times 4n} & 0_{n\times n} & 0_{n\times(l+4)n} \\ 0_{n\times mn} & 0_{n\times n} & 0_{n\times 4n} & I_{n\times n} & 0_{n\times(l+4)n} \end{pmatrix};$$

$$W_{\tilde{R}}=\begin{pmatrix} 0_{n\times n} & 0_{n\times(m+2)n} & \sqrt{\frac{\tau_1}{m}}I_{n\times n} & 0_{n\times n} & 0_{n\times 2n} & 0_{n\times n} & 0_{n\times ln} & 0_{n\times n} & 0_{n\times n} \\ \sqrt{\frac{\tau_1}{m}}I_{n\times n} & 0_{n\times(m+2)n} & 0_{n\times n} & 0_{n\times n} & 0_{n\times 2n} & 0_{n\times n} & 0_{n\times ln} & 0_{n\times n} & 0_{n\times n} \\ 0_{n\times n} & 0_{n\times(m+2)n} & \sqrt{\tau_2-\tau_1}I_{n\times n} & 0_{n\times n} & 0_{n\times 2n} & 0_{n\times n} & 0_{n\times ln} & 0_{n\times n} & 0_{n\times n} \\ 0_{n\times n} & 0_{n\times(m+2)n} & 0_{n\times n} & \sqrt{\frac{r}{l}}I_{n\times n} & 0_{n\times 2n} & 0_{n\times n} & 0_{n\times ln} & 0_{n\times n} & 0_{n\times n} \\ 0_{n\times n} & 0_{n\times(m+2)n} & 0_{n\times n} & 0_{n\times n} & 0_{n\times 2n} & 0_{n\times n} & 0_{n\times ln} & \sqrt{\frac{m}{\tau_1}}I_{n\times n} & 0_{n\times n} \\ 0_{n\times n} & 0_{n\times(m+2)n} & 0_{n\times n} & 0_{n\times n} & 0_{n\times 2n} & \sqrt{\frac{l}{r}}I_{n\times n} & 0_{n\times ln} & 0_{n\times n} & 0_{n\times n} \end{pmatrix};$$

$$W_{\tilde{S}}=\begin{pmatrix} 0_{n\times(m+3)n} & \frac{1}{\sqrt{2}}\frac{\tau_1}{m}I_{n\times n} & 0_{n\times n} & 0_{n\times(l+4)n} & 0_{n\times n} \\ 0_{n\times(m+3)n} & 0_{n\times n} & \frac{r}{\sqrt{2}l}I_{n\times n} & 0_{n\times(l+4)n} & 0_{n\times n} \\ 0_{n\times(m+3)n} & 0_{n\times n} & 0_{n\times n} & 0_{n\times(l+4)n} & \frac{\sqrt{2}l}{r}I_{n\times n} \end{pmatrix};$$

$\tilde{\Lambda}_1=\begin{pmatrix} -\Lambda_1L_3 & \Lambda_1L_4 \\ * & -\Lambda_1 \end{pmatrix}$；$\tilde{\Lambda}_2=\begin{pmatrix} -\Lambda_2L_3 & \Lambda_2L_4 \\ * & -\Lambda_2 \end{pmatrix}$；

$\tilde{Q}_3=\mathrm{diag}(Q_3,-(1-\tau_3)Q_3)$；$\tilde{Z}=\mathrm{diag}(Z,-Z)$；

$\tilde{R}=\mathrm{diag}(R_1,R_2,R_3,R_4,-R_2,-R_4)$；

$\tilde{S}=\mathrm{diag}(S_1,S_2,-S_2)$；

$\tilde{M}^{\mathrm{T}}=\left(M_1^{\mathrm{T}},M_2^{\mathrm{T}},0_{n\times(m+l+8)n}\right)$；

$\hat{M}=\left(I_{n\times n},-I_{n\times n},0_{n\times(m+l+8)n}\right)$；

$\tilde{N}^{\mathrm{T}}=\left(0_{n\times mn},N_1^{\mathrm{T}},0_{n\times n},N_2^{\mathrm{T}},0_{n\times(l+7)n}\right)$；

$\hat{N}=\left(0_{n\times mn},I_{n\times n},0_{n\times n},-I_{n\times n},0_{n\times(l+7)n}\right)$；

$\tilde{F}^{\mathrm{T}}=\left(F_1^{\mathrm{T}},0_{n\times(m+l+7)n},F_2^{\mathrm{T}},0_{n\times n}\right)$；

$\hat{F}=\left(\frac{\tau_1}{m}I_{n\times n},0_{n\times(m+l+7)n},-I_{n\times n},0_{n\times n}\right)$；

$\tilde{G}^{\mathrm{T}}=\left(G_1^{\mathrm{T}},0_{n\times(m+2)n},G_2^{\mathrm{T}},G_3^{\mathrm{T}},0_{n\times(l+5)n}\right)$；

$\hat{G}=\left(-C,0_{n\times(m+2)n},-I_{n\times n},A,0_{n\times n},B,D,D,\cdots,D,0_{n\times 3n}\right)$。

证明：由Brouwer不动点定理，不难证明系统平衡点的存在性，这里仅需证明平衡点的全局渐近稳定性。

为此，构造如下形式的增广Lyapunov-Krasovskii泛函 $V(t)=V_1(t)+V_2(t)+V_3(t)+V_4(t)$。

式中：

$$V_1(t)=\varsigma^{\mathrm{T}}(t)P\varsigma(t)+2\sum_{j=1}^{n}\left(d_j\int_0^{x_j(t)}\left(f_j(s)-l_j^- s\right)\mathrm{d}s+e_j\int_0^{x_j(t)}\left(l_j^+ s-f_j(s)\right)\mathrm{d}s\right);$$

$$V_2(t)=\int_{t-\frac{\tau_1}{m}}^{t}\eta_1^{\mathrm{T}}(s)Q_1\eta_1(s)\mathrm{d}s+\int_{t-\frac{r}{l}}^{t}\eta_2^{\mathrm{T}}(s)Q_2\eta_2(s)\mathrm{d}s+\int_{t-\tau(t)}^{t-\tau_1}\eta_3^{\mathrm{T}}(s)Q_3\eta_3(s)\mathrm{d}s+\int_{t-\tau_2}^{t-\tau_1}x^{\mathrm{T}}(s)Zx(s)\mathrm{d}s;$$

$V_3(t)=\int_{-\frac{\tau_1}{m}}^{0}\int_{t+\theta}^{t}\dot{\boldsymbol{x}}^{\mathrm{T}}(s)\boldsymbol{R}_1\dot{\boldsymbol{x}}(s)\mathrm{d}s\mathrm{d}\theta+\int_{-\frac{\tau_1}{m}}^{0}\int_{t+\theta}^{t}\boldsymbol{x}^{\mathrm{T}}(s)\boldsymbol{R}_2\boldsymbol{x}(s)\mathrm{d}s\mathrm{d}\theta+\int_{-\tau_2}^{-\tau_1}\int_{t+\theta}^{t}\dot{\boldsymbol{x}}^{\mathrm{T}}(s)\boldsymbol{R}_3\dot{\boldsymbol{x}}(s)\mathrm{d}s\mathrm{d}\theta+\int_{-\frac{r}{l}}^{0}\int_{t+\theta}^{t}\boldsymbol{f}^{\mathrm{T}}\big(\boldsymbol{x}(s)\big)\boldsymbol{R}_4\boldsymbol{f}\big(\boldsymbol{x}(s)\big)\mathrm{d}s\mathrm{d}\theta$;

$V_4(t)=\int_{-\frac{\tau_1}{m}}^{0}\int_{\theta}^{0}\int_{t+\lambda}^{t}\dot{\boldsymbol{x}}^{\mathrm{T}}(s)\boldsymbol{S}_1\dot{\boldsymbol{x}}(s)\mathrm{d}s\mathrm{d}\lambda\mathrm{d}\theta+\int_{-\frac{r}{l}}^{0}\int_{\theta}^{0}\int_{t+\lambda}^{t}\boldsymbol{f}^{\mathrm{T}}\big(\boldsymbol{x}(s)\big)\boldsymbol{S}_2\boldsymbol{f}\big(\boldsymbol{x}(s)\big)\mathrm{d}s\mathrm{d}\lambda\mathrm{d}\theta$。

现计算 $V(t)$ 沿系统(2)对时间的导数,并进行适当的放大整理。

$$\dot{V}_1(t)=2\boldsymbol{\varsigma}^{\mathrm{T}}(t)\boldsymbol{P}\dot{\boldsymbol{\varsigma}}(t)+2\Big(\boldsymbol{f}^{\mathrm{T}}\big(\boldsymbol{x}(t)\big)-\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{L}_1\Big)\boldsymbol{W}\dot{\boldsymbol{x}}(t)+2\Big(\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{L}_2-\boldsymbol{f}^{\mathrm{T}}\big(\boldsymbol{x}(t)\big)\Big)\boldsymbol{\Delta}\dot{\boldsymbol{x}}(t)\ , \tag{5}$$

$$\begin{aligned}\dot{V}_2(t)\leqslant{}&\boldsymbol{\eta}_1^{\mathrm{T}}(t)\boldsymbol{Q}_1\boldsymbol{\eta}_1(t)-\boldsymbol{\eta}_1^{\mathrm{T}}\Big(t-\frac{\tau_1}{m}\Big)\boldsymbol{Q}_1\boldsymbol{\eta}_1\Big(t-\frac{\tau_1}{m}\Big)+\boldsymbol{\eta}_2^{\mathrm{T}}(t)\boldsymbol{Q}_2\boldsymbol{\eta}_2(t)-\boldsymbol{\eta}_2^{\mathrm{T}}\Big(t-\frac{r}{l}\Big)\boldsymbol{Q}_2\cdot\\&\boldsymbol{\eta}_3^{\mathrm{T}}(t-\tau_1)\boldsymbol{Q}_3\boldsymbol{\eta}_3(t-\tau_1)-(1-\tau_3)\boldsymbol{\eta}_3^{\mathrm{T}}\big(t-\tau(t)\big)\boldsymbol{Q}_3\boldsymbol{\eta}_3\big(t-\tau(t)\big)+\boldsymbol{x}^{\mathrm{T}}(t-\tau_1)\boldsymbol{Z}x(t-\tau_1)-\boldsymbol{x}^{\mathrm{T}}(t-\tau_2)\boldsymbol{Z}x(t-\tau_2)\ ,\end{aligned} \tag{6}$$

$$\begin{aligned}\dot{V}_3(t)={}&\frac{\tau_1}{m}\dot{\boldsymbol{x}}^{\mathrm{T}}(t)\boldsymbol{R}_1\dot{\boldsymbol{x}}(t)-\int_{t-\frac{\tau_1}{m}}^{t}\dot{\boldsymbol{x}}^{\mathrm{T}}(s)\boldsymbol{R}_1\dot{\boldsymbol{x}}(s)\mathrm{d}s+\frac{\tau_1}{m}\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{R}_2\boldsymbol{x}(t)-\int_{t-\frac{\tau_1}{m}}^{t}\boldsymbol{x}^{\mathrm{T}}(s)\boldsymbol{R}_2\boldsymbol{x}(s)\mathrm{d}s+\\&(\tau_2-\tau_1)\dot{\boldsymbol{x}}^{\mathrm{T}}(t)\boldsymbol{R}_3\dot{\boldsymbol{x}}(t)-\int_{t-\tau_2}^{t-\tau_1}\dot{\boldsymbol{x}}^{\mathrm{T}}(s)\boldsymbol{R}_3\dot{\boldsymbol{x}}(s)\mathrm{d}s+\frac{r}{l}\boldsymbol{f}^{\mathrm{T}}\big(\boldsymbol{x}(t)\big)\boldsymbol{R}_4\boldsymbol{f}\big(\boldsymbol{x}(t)\big)-\int_{t-\frac{r}{l}}^{t}\boldsymbol{f}^{\mathrm{T}}\big(\boldsymbol{x}(s)\big)\boldsymbol{R}_4\boldsymbol{f}\big(\boldsymbol{x}(s)\big)\mathrm{d}s\ ,\end{aligned} \tag{7}$$

$$\dot{V}_4(t)=\frac{1}{2}\Big(\frac{\tau_1}{m}\Big)^2\dot{\boldsymbol{x}}^{\mathrm{T}}(t)\boldsymbol{S}_1\dot{\boldsymbol{x}}(t)-\int_{-\frac{\tau_1}{m}}^{0}\int_{t+\theta}^{t}\dot{\boldsymbol{x}}^{\mathrm{T}}(s)\boldsymbol{S}_1\dot{\boldsymbol{x}}(s)\mathrm{d}s\mathrm{d}\theta+\frac{1}{2}\Big(\frac{r}{l}\Big)^2\boldsymbol{f}^{\mathrm{T}}\big(\boldsymbol{x}(t)\big)\boldsymbol{S}_2\boldsymbol{f}\big(\boldsymbol{x}(t)\big)-\int_{-\frac{r}{l}}^{0}\int_{t+\theta}^{t}\boldsymbol{f}^{\mathrm{T}}\big(\boldsymbol{x}(s)\big)\boldsymbol{S}_2\boldsymbol{f}\big(\boldsymbol{x}(s)\big)\mathrm{d}s\mathrm{d}\theta\ 。 \tag{8}$$

对于 $\dot{V}_3(t)$ 、$\dot{V}_4(t)$ 中如下的3个积分项,由引理1,有:

$$-\int_{t-\frac{\tau_1}{m}}^{t}\boldsymbol{x}^{\mathrm{T}}(s)\boldsymbol{R}_2\boldsymbol{x}(s)\mathrm{d}s\leqslant-\frac{m}{\tau_1}\bigg(\int_{t-\frac{\tau_1}{m}}^{t}\boldsymbol{x}(s)\mathrm{d}s\bigg)^{\mathrm{T}}\boldsymbol{R}_2\bigg(\int_{t-\frac{\tau_1}{m}}^{t}\boldsymbol{x}(s)\mathrm{d}s\bigg)\ , \tag{9}$$

$$-\int_{t-\frac{r}{l}}^{t}\boldsymbol{f}^{\mathrm{T}}\big(\boldsymbol{x}(s)\big)\boldsymbol{R}_4\boldsymbol{f}\big(\boldsymbol{x}(s)\big)\mathrm{d}s\leqslant-\frac{l}{r}\bigg(\int_{t-\frac{r}{l}}^{t}\boldsymbol{f}\big(\boldsymbol{x}(s)\big)\mathrm{d}s\bigg)^{\mathrm{T}}\boldsymbol{R}_4\bigg(\int_{t-\frac{r}{l}}^{t}\boldsymbol{f}\big(\boldsymbol{x}(s)\big)\mathrm{d}s\bigg)\ , \tag{10}$$

$$-\int_{-\frac{r}{l}}^{0}\int_{t+\theta}^{t}\boldsymbol{f}^{\mathrm{T}}\big(\boldsymbol{x}(s)\big)\boldsymbol{S}_2\boldsymbol{f}\big(\boldsymbol{x}(s)\big)\mathrm{d}s\mathrm{d}\theta\leqslant-2\Big(\frac{l}{r}\Big)^2\bigg(\int_{-\frac{r}{l}}^{0}\int_{t+\theta}^{t}\boldsymbol{f}\big(\boldsymbol{x}(s)\big)\mathrm{d}s\mathrm{d}\theta\bigg)^{\mathrm{T}}\boldsymbol{S}_2\bigg(\int_{-\frac{r}{l}}^{0}\int_{t+\theta}^{t}\boldsymbol{f}\big(\boldsymbol{x}(s)\big)\mathrm{d}s\mathrm{d}\theta\bigg)\ 。 \tag{11}$$

根据Newton-Leibniz公式及系统(2),对于恰当维数的矩阵 $\boldsymbol{M}_i$、$\boldsymbol{N}_i$、$\boldsymbol{F}_i$、$\boldsymbol{G}_j$($i=1,2;j=1,2,3$),总有如下的等式成立。

$$2\bigg(\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{M}_1+\boldsymbol{x}^{\mathrm{T}}\Big(t-\frac{\tau_1}{m}\Big)\boldsymbol{M}_2\bigg)\bigg(\boldsymbol{x}(t)-\boldsymbol{x}\Big(t-\frac{\tau_1}{m}\Big)-\int_{t-\frac{\tau_1}{m}}^{t}\dot{\boldsymbol{x}}(s)\mathrm{d}s\bigg)=0\ ; \tag{12}$$

$$2\Big(\boldsymbol{x}^{\mathrm{T}}(t-\tau_1)\boldsymbol{N}_1+\boldsymbol{x}^{\mathrm{T}}(t-\tau_2)\boldsymbol{N}_2\Big)\Big(\boldsymbol{x}(t-\tau_1)-\boldsymbol{x}(t-\tau_2)-\int_{t-\tau_2}^{t-\tau_1}\dot{\boldsymbol{x}}(s)\mathrm{d}s\Big)=0\ ; \tag{13}$$

$$2\bigg(\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{F}_1+\int_{t-\frac{\tau_1}{m}}^{t}\boldsymbol{x}^{\mathrm{T}}(s)\mathrm{d}s\boldsymbol{F}_2\bigg)\bigg(\frac{\tau_1}{m}\boldsymbol{x}(t)-\int_{t-\frac{\tau_1}{m}}^{t}\boldsymbol{x}(s)\mathrm{d}s-\int_{-\frac{\tau_1}{m}}^{0}\int_{t+\theta}^{t}\dot{\boldsymbol{x}}(s)\mathrm{d}s\mathrm{d}\theta\bigg)=0\ ; \tag{14}$$

$$2\Big(\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{G}_1+\dot{\boldsymbol{x}}^{\mathrm{T}}(t)\boldsymbol{G}_2+\boldsymbol{f}^{\mathrm{T}}\big(\boldsymbol{x}(t)\big)\boldsymbol{G}_3\Big)\cdot\Big(-\boldsymbol{C}\boldsymbol{x}(t)+\boldsymbol{A}\boldsymbol{f}\big(\boldsymbol{x}(t)\big)+\boldsymbol{B}\boldsymbol{f}\Big(\boldsymbol{x}\big(t-\tau(t)\big)\Big)+\boldsymbol{D}\int_{t-r}^{t}\boldsymbol{f}\big(\boldsymbol{x}(s)\big)\mathrm{d}s-\dot{\boldsymbol{x}}(t)\Big)=0\ 。 \tag{15}$$

对于式(12)~(14)中关于 $\dot{\boldsymbol{x}}(s)$ 的积分项,由 $\boldsymbol{\xi}(t)$ 的定义及均值不等式,易得:

$$-2\boldsymbol{\xi}^{\mathrm{T}}(t)\tilde{\boldsymbol{M}}\int_{t-\frac{\tau_1}{m}}^{t}\dot{\boldsymbol{x}}(s)\mathrm{d}s\leqslant\frac{\tau_1}{m}\boldsymbol{\xi}^{\mathrm{T}}(t)\tilde{\boldsymbol{M}}\boldsymbol{R}_1^{-1}\tilde{\boldsymbol{M}}\boldsymbol{\xi}(t)+\int_{t-\frac{\tau_1}{m}}^{t}\dot{\boldsymbol{x}}^{\mathrm{T}}(s)\boldsymbol{R}_1\dot{\boldsymbol{x}}(s)\mathrm{d}s\ ; \tag{16}$$

$$-2\boldsymbol{\xi}^{\mathrm{T}}(t)\tilde{\boldsymbol{N}}\int_{t-\tau_2}^{t-\tau_1}\dot{\boldsymbol{x}}(s)\mathrm{d}s\leqslant(\tau_2-\tau_1)\boldsymbol{\xi}^{\mathrm{T}}(t)\tilde{\boldsymbol{N}}\boldsymbol{R}_3^{-1}\tilde{\boldsymbol{N}}^{\mathrm{T}}\boldsymbol{\xi}(t)+\int_{t-\tau_2}^{t-\tau_1}\dot{\boldsymbol{x}}^{\mathrm{T}}(s)\boldsymbol{R}_3\dot{\boldsymbol{x}}(s)\mathrm{d}s\ ; \tag{17}$$

$$-2\boldsymbol{\xi}^{\mathrm{T}}(t)\tilde{\boldsymbol{F}}\int_{-\frac{\tau_1}{m}}^{0}\int_{t+\theta}^{t}\dot{\boldsymbol{x}}(s)\mathrm{d}s\mathrm{d}\theta\leqslant\frac{1}{2}\Big(\frac{\tau_1}{m}\Big)^2\boldsymbol{\xi}^{\mathrm{T}}(t)\tilde{\boldsymbol{F}}\boldsymbol{S}_1^{-1}\tilde{\boldsymbol{F}}^{\mathrm{T}}\boldsymbol{\xi}(t)+\int_{-\frac{\tau_1}{m}}^{0}\int_{t+\theta}^{t}\dot{\boldsymbol{x}}^{\mathrm{T}}(s)\boldsymbol{S}_1\dot{\boldsymbol{x}}(s)\mathrm{d}s\mathrm{d}\theta\ 。 \tag{18}$$

由关于激励函数的假设H1,对任意非负标量 $\lambda_{ij}\geqslant0,\ i=1,2;j=1,2,\cdots,n$,下面的不等式成立:

$$2\sum_{j=1}^{n}\lambda_{1j}\left(f_j\left(x_j(t)\right)-l_j^- x_j(t)\right)\left(l_j^+ x_j(t)-f_j\left(x_j(t)\right)\right)\geqslant 0\ ;$$

$$2\sum_{j=1}^{n}\lambda_{1j}\left(f_j\left(x_j(t-\tau)\right)-l_j^- x_j(t-\tau)\right)\left(l_j^+ x_j(t-\tau)-f_j\left(x_j(t-\tau)\right)\right)\geqslant 0\ 。$$

或由矩阵向量形式等价地描述为：

$$\left(\boldsymbol{x}^{\mathrm{T}}(t),\boldsymbol{f}^{\mathrm{T}}\left(\boldsymbol{x}(t)\right)\right)\begin{pmatrix}-\boldsymbol{\Lambda}_1\boldsymbol{L}_3 & \boldsymbol{\Lambda}_1\boldsymbol{L}_4\\ * & -\boldsymbol{\Lambda}_1\end{pmatrix}\begin{pmatrix}\boldsymbol{x}(t)\\ \boldsymbol{f}\left(\boldsymbol{x}(t)\right)\end{pmatrix}\geqslant 0\ , \tag{19}$$

$$\left(\boldsymbol{x}^{\mathrm{T}}(t-\tau_1),\boldsymbol{f}^{\mathrm{T}}\left(\boldsymbol{x}(t-\tau_1)\right)\right)\begin{pmatrix}-\boldsymbol{\Lambda}_2\boldsymbol{L}_3 & \boldsymbol{\Lambda}_2\boldsymbol{L}_4\\ * & -\boldsymbol{\Lambda}_2\end{pmatrix}\begin{pmatrix}\boldsymbol{x}(t-\tau_1)\\ \boldsymbol{f}\left(\boldsymbol{x}(t-\tau_1)\right)\end{pmatrix}\geqslant 0\ 。 \tag{20}$$

由式(5)~(20)，经过一系列运算、整理，不难得到：

$$\dot{V}(t)\leqslant\boldsymbol{\xi}^{\mathrm{T}}(t)\left(\boldsymbol{\Xi}+\frac{\tau_1}{m}\tilde{\boldsymbol{M}}\boldsymbol{R}_1^{-1}\tilde{\boldsymbol{M}}^{\mathrm{T}}+(\tau_2-\tau_1)\tilde{\boldsymbol{N}}\boldsymbol{R}_3^{-1}\tilde{\boldsymbol{N}}^{\mathrm{T}}+\frac{1}{2}\left(\frac{\tau_1}{m}\right)^2\tilde{\boldsymbol{F}}\boldsymbol{S}_1^{-1}\tilde{\boldsymbol{F}}^{\mathrm{T}}\right)\boldsymbol{\xi}(t)\ , \tag{21}$$

其中：

$$\boldsymbol{\xi}^{\mathrm{T}}(t)=\Big(\boldsymbol{\eta}_1^{\mathrm{T}}(t),\boldsymbol{x}^{\mathrm{T}}(t-\tau_1),\boldsymbol{x}^{\mathrm{T}}(t-\tau(t)),\boldsymbol{x}^{\mathrm{T}}(t-\tau_2),\dot{\boldsymbol{x}}^{\mathrm{T}}(t),\boldsymbol{f}^{\mathrm{T}}\left(\boldsymbol{x}(t)\right),\boldsymbol{f}^{\mathrm{T}}\left(\boldsymbol{x}(t-\tau_1)\right),\boldsymbol{f}^{\mathrm{T}}\left(\boldsymbol{x}(t-\tau(t))\right),\boldsymbol{\eta}_2^{\mathrm{T}}(t),\int_{t-\frac{l+1}{l}r}^{t-r}\boldsymbol{f}^{\mathrm{T}}\left(\boldsymbol{x}(s)\right)\mathrm{d}s,$$

$$\int_{t-\frac{\tau_1}{m}}^{t}\boldsymbol{x}^{\mathrm{T}}(s)\mathrm{d}s\int_{-\frac{r}{l}}^{0}\int_{t+\theta}^{t}\boldsymbol{f}^{\mathrm{T}}\left(\boldsymbol{x}(s)\right)\mathrm{d}s\mathrm{d}\theta\Big)$$ 为 $(m+l+10)n$ 维的行向量。

由系统(3)以及引理2易知，存在恰当的小正数 ε，使得 $\boldsymbol{\Sigma}<\mathrm{diag}(-\varepsilon\boldsymbol{I},\boldsymbol{0},\boldsymbol{0},\cdots,\boldsymbol{0})$，其中：

$$\boldsymbol{\Sigma}=\boldsymbol{\Xi}+\frac{\tau_1}{m}\tilde{\boldsymbol{M}}\boldsymbol{R}_1^{-1}\tilde{\boldsymbol{M}}^{\mathrm{T}}+(\tau_2-\tau_1)\tilde{\boldsymbol{N}}\boldsymbol{R}_3^{-1}\tilde{\boldsymbol{N}}^{\mathrm{T}}+\frac{1}{2}\left(\frac{\tau_1}{m}\right)^2\tilde{\boldsymbol{F}}\boldsymbol{S}_1^{-1}\tilde{\boldsymbol{F}}^{\mathrm{T}}\ 。$$

于是，有 $\dot{V}(t)\leqslant\boldsymbol{\xi}^{\mathrm{T}}(t)\boldsymbol{\Sigma}\boldsymbol{\xi}(t)<-\varepsilon\left\|\boldsymbol{x}(t)\right\|^2$ 。

这意味着时滞系统(2)的平衡点是全局渐近稳定的，证毕。

注1：正如前文所说，时滞分割技术被广泛应用于定常或时变的离散时滞系统，却很少应用于分布时滞系统。受文献[18]的启发，本文首次尝试将时滞分割技术推广应用于同时具有时变时滞和分布时滞的神经网络系统的稳定性分析。

注2：定理1中，稳定性条件保守性的降低得益于新构造的增广Lyapunov-Krasovskii泛函，其中不仅含有针对时变时滞和分布时滞的时滞分割积分项，也含有时滞分割的二重及三重积分项，如 $\boldsymbol{V}_2(t)$ 中的前2个积分项，$\boldsymbol{V}_3(t)$ 中的前2个二重积分项和 $\boldsymbol{V}_4(t)$ 中的三重积分项。

注3：在文献[20]中，Chen等通过构造含有一个Lyapunov-Krasovskii二重及三重积分项的增广泛函，未使用时滞分割技术研究了一类具有离散时滞的神经网络系统的稳定性条件。而基于时滞分割技术的新的Lyapunov-Krasovskii泛函却可以获得比已有文献结果具有更低保守性的系统稳定性条件。

注4：在文献[13]中给出了具有离散区间时滞和分布时滞的随机神经网络系统的指数稳定性条件，值得注意的是文中的分布时滞是时变的。本文虽然考虑的是定常分布时滞，但本文的方法能够推广到时变分布时滞系统上去，这也是近期将开展的工作。

下面将考虑系统(2)的2个特殊情形：

情形1：若 $\tau(t)=\tau$，则经过平衡点平移变换后，相应的系统由下式给出

$$\dot{\boldsymbol{x}}(t)=-\boldsymbol{C}\boldsymbol{x}(t)+\boldsymbol{A}\boldsymbol{f}\left(\boldsymbol{x}(t)\right)+\boldsymbol{B}\boldsymbol{f}\left(\boldsymbol{x}(t-\tau)\right)+\boldsymbol{D}\int_{t-r}^{t}\boldsymbol{f}\left(\boldsymbol{x}(s)\right)\mathrm{d}s\ 。 \tag{22}$$

对上述系统构造Lyapunov-Krasovskii泛函

$$V(t)=V_1(t)+V_2(t)+V_3(t)+V_4(t)\ ,$$

式中：

$$V_1(t)=\boldsymbol{\varsigma}^{\mathrm{T}}(t)\boldsymbol{P}\boldsymbol{\varsigma}(t)+2\sum_{j=1}^{n}\left(d_j\int_0^{x_j(t)}\left(f_j(s)-l_j^- s\right)\mathrm{d}s+e_j\int_0^{x_j(t)}\left(l_j^+ s-f_j(s)\right)\mathrm{d}s\right)\ ;$$

$$V_2(t)=\int_{t-\frac{\tau}{m}}^{t}\boldsymbol{\eta}_1^{\mathrm{T}}(s)\boldsymbol{Q}_1\boldsymbol{\eta}_1(s)\mathrm{d}s+\int_{t-\frac{r}{l}}^{t}\boldsymbol{\eta}_2^{\mathrm{T}}(s)\boldsymbol{Q}_2\boldsymbol{\eta}_2(s)\mathrm{d}s+\int_{t-\tau}^{t}\boldsymbol{\eta}_3^{\mathrm{T}}(s)\boldsymbol{Q}_3\boldsymbol{\eta}_3(s)\mathrm{d}s\ ;$$

$$V_3(t)=\int_{-\frac{\tau}{m}}^{0}\int_{t+\theta}^{t}\dot{\boldsymbol{x}}^{\mathrm{T}}(s)\boldsymbol{R}_1\dot{\boldsymbol{x}}(s)\mathrm{d}s\mathrm{d}\theta+\int_{-\frac{\tau}{m}}^{0}\int_{t+\theta}^{t}\boldsymbol{x}^{\mathrm{T}}(s)\boldsymbol{R}_2\boldsymbol{x}(s)\mathrm{d}s\mathrm{d}\theta+\int_{-\tau}^{0}\int_{t+\theta}^{t}\dot{\boldsymbol{x}}^{\mathrm{T}}(s)\boldsymbol{R}_3\dot{\boldsymbol{x}}(s)\mathrm{d}s\mathrm{d}\theta+\int_{-\frac{r}{l}}^{0}\int_{t+\theta}^{t}\boldsymbol{f}^{\mathrm{T}}\left(\boldsymbol{x}(s)\right)\boldsymbol{R}_4\boldsymbol{f}\left(\boldsymbol{x}(s)\right)\mathrm{d}s\mathrm{d}\theta\ ;$$

$$V_4(t)=\int_{-\frac{\tau}{m}}^{0}\int_{\theta}^{0}\int_{t+\lambda}^{t}\dot{\boldsymbol{x}}^{\mathrm{T}}(s)\boldsymbol{S}_1\dot{\boldsymbol{x}}(s)\mathrm{d}s\mathrm{d}\lambda\mathrm{d}\theta+\int_{-\frac{r}{l}}^{0}\int_{\theta}^{0}\int_{t+\lambda}^{t}\boldsymbol{f}^{\mathrm{T}}\left(\boldsymbol{x}(s)\right)\boldsymbol{S}_2\boldsymbol{f}\left(\boldsymbol{x}(s)\right)\mathrm{d}s\mathrm{d}\lambda\mathrm{d}\theta\ 。$$

通过类似定理1的方法,容易得到如下推论。

推论1:对给定的τ、r,正整数$m,l\geqslant 1$,若存在对称正定阵$\boldsymbol{P}\in\mathbb{R}^{3n\times 3n}$,$\boldsymbol{Q}_1\in\mathbb{R}^{mn\times mn}$,$\boldsymbol{Q}_2\in\mathbb{R}^{ln\times ln}$,$\boldsymbol{Q}_3\in\mathbb{R}^{2n\times 2n}$,$\boldsymbol{R}_i\in\mathbb{R}^{n\times n}(i=1,2,3,4)$,$\boldsymbol{S}_i\in\mathbb{R}^{n\times n}(i=1,2)$,非负对角阵$\boldsymbol{W}=\mathrm{diag}(w_1,w_2,\cdots,w_n)$,$\boldsymbol{\Delta}=\mathrm{diag}(\delta_1,\delta_2,\cdots,\delta_n)$,$\boldsymbol{\Lambda}_i=\mathrm{diag}(\lambda_{i1},\lambda_{i2},\cdots,\lambda_{in}),i=1,2$以及任意恰当维数的矩阵$\boldsymbol{M}_i$、$\boldsymbol{N}_i$、$\boldsymbol{F}_i$、$\boldsymbol{G}_j$($i=1,2;j=1,2,3,4$)使如下的LMI成立,则系统(22)的平衡点全局渐近稳定,

$$\begin{pmatrix}\boldsymbol{\Sigma} & \sqrt{\dfrac{\tau}{m}}\tilde{\boldsymbol{M}}' & \sqrt{\tau}\tilde{\boldsymbol{N}}' & \dfrac{1}{\sqrt{2}}\dfrac{\tau}{m}\tilde{\boldsymbol{F}}' \\ * & -\boldsymbol{R}_1 & \boldsymbol{0} & \boldsymbol{0} \\ * & * & -\boldsymbol{R}_3 & \boldsymbol{0} \\ * & * & * & -\boldsymbol{S}_1\end{pmatrix}<0,$$

式中,$\boldsymbol{\Sigma}$定义为:

$$\boldsymbol{\Sigma}=\mathrm{sym}\left(\boldsymbol{\Sigma}_{P_1}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{\Sigma}_{P_2}\right)+\mathrm{sym}\left(\boldsymbol{\Sigma}_w^{\mathrm{T}}\tilde{\boldsymbol{W}}\boldsymbol{\Sigma}_E\right)+\boldsymbol{\Sigma}_{Q_1}^{\mathrm{T}}\boldsymbol{Q}_1\boldsymbol{\Sigma}_{Q_1}-\boldsymbol{\Sigma}_{Q'_1}^{\mathrm{T}}\boldsymbol{Q}_1\boldsymbol{\Sigma}_{Q'_1}+\boldsymbol{\Sigma}_{Q_2}^{\mathrm{T}}\boldsymbol{Q}_2\boldsymbol{\Sigma}_{Q_2}-\boldsymbol{\Sigma}_{Q'_2}^{\mathrm{T}}\boldsymbol{Q}_2\boldsymbol{\Sigma}_{Q'_2}+\boldsymbol{\Sigma}_{\tilde{Q}_3}^{\mathrm{T}}\tilde{\boldsymbol{Q}}_3\boldsymbol{\Sigma}_{\tilde{Q}_3}+\boldsymbol{\Sigma}_{\tilde{R}}^{\mathrm{T}}\tilde{\boldsymbol{R}}\boldsymbol{\Sigma}_{\tilde{R}}+$$
$$\boldsymbol{\Sigma}_{\tilde{S}}^{\mathrm{T}}\tilde{\boldsymbol{S}}\boldsymbol{\Sigma}_{\tilde{S}}+\mathrm{sym}\left(\tilde{\boldsymbol{M}}'\hat{\boldsymbol{M}}'\right)+\mathrm{sym}\left(\tilde{\boldsymbol{N}}'\hat{\boldsymbol{N}}\right)+\mathrm{sym}\left(\tilde{\boldsymbol{F}}'\hat{\boldsymbol{F}}'\right)+\mathrm{sym}\left(\tilde{\boldsymbol{G}}'\hat{\boldsymbol{G}}'\right)+\boldsymbol{\Sigma}_{\tilde{\Lambda}_1}^{\mathrm{T}}\tilde{\boldsymbol{\Lambda}}_1\boldsymbol{\Sigma}_{\tilde{\Lambda}_1}+\boldsymbol{\Sigma}_{\tilde{\Lambda}_2}^{\mathrm{T}}\tilde{\boldsymbol{\Lambda}}_2\boldsymbol{\Sigma}_{\tilde{\Lambda}_2}。\tag{23}$$

式(23)中:

$$\boldsymbol{\Sigma}_{P_1}=\begin{pmatrix}\boldsymbol{I}_{n\times n} & \boldsymbol{0}_{n\times(m+l+4)n} & \boldsymbol{0}_{n\times n} & \boldsymbol{0}_{n\times n}\\ \boldsymbol{0}_{n\times n} & \boldsymbol{0}_{n\times(m+l+4)n} & \boldsymbol{I}_{n\times n} & \boldsymbol{0}_{n\times n}\\ \boldsymbol{0}_{n\times n} & \boldsymbol{0}_{n\times(m+l+4)n} & \boldsymbol{0}_{n\times n} & \boldsymbol{I}_{n\times n}\end{pmatrix};\ \boldsymbol{\Sigma}_{P_2}=\begin{pmatrix}\boldsymbol{0}_{n\times n} & \boldsymbol{0}_{n\times n} & \boldsymbol{0}_{n\times(m-1)n} & \boldsymbol{I}_{n\times n} & \boldsymbol{0}_{n\times n} & \boldsymbol{0}_{n\times n} & \boldsymbol{0}_{n\times n} & \boldsymbol{0}_{n\times(l+2)n}\\ \boldsymbol{I}_{n\times n} & -\boldsymbol{I}_{n\times n} & \boldsymbol{0}_{n\times(m-1)n} & \boldsymbol{0}_{n\times n} & \boldsymbol{0}_{n\times n} & \boldsymbol{0}_{n\times n} & \boldsymbol{0}_{n\times n} & \boldsymbol{0}_{n\times(l+2)n}\\ \boldsymbol{0}_{n\times n} & \boldsymbol{0}_{n\times n} & \boldsymbol{0}_{n\times(m-1)n} & \boldsymbol{0}_{n\times n} & \dfrac{r}{l}\boldsymbol{I}_{n\times n} & \boldsymbol{0}_{n\times n} & -\boldsymbol{I}_{n\times n} & \boldsymbol{0}_{n\times(l+2)n}\end{pmatrix};$$

$$\boldsymbol{\Sigma}_W=\begin{pmatrix}-\boldsymbol{L}_1 & \boldsymbol{0}_{n\times(m+1)n} & \boldsymbol{I}_{n\times n} & \boldsymbol{0}_{n\times(l+4)n}\\ \boldsymbol{L}_2 & \boldsymbol{0}_{n\times(m+1)n} & -\boldsymbol{I}_{n\times n} & \boldsymbol{0}_{n\times(l+4)n}\end{pmatrix};\ \tilde{\boldsymbol{W}}=\begin{pmatrix}\boldsymbol{W} & \\ & \boldsymbol{\Delta}\end{pmatrix};\ \boldsymbol{\Sigma}_E=\begin{pmatrix}\boldsymbol{0}_{n\times(m+1)n} & \boldsymbol{I}_{n\times n} & \boldsymbol{0}_{n\times(l+5)n}\\ \boldsymbol{0}_{n\times(m+1)n} & \boldsymbol{I}_{n\times n} & \boldsymbol{0}_{n\times(l+5)n}\end{pmatrix};\ \boldsymbol{\Sigma}_{Q_1}=\begin{pmatrix}\boldsymbol{I}_{mn\times mn} & \boldsymbol{0}_{mn\times(l+7)n}\end{pmatrix};$$

$$\boldsymbol{\Sigma}_{Q'_1}=\begin{pmatrix}\boldsymbol{0}_{mn\times n} & \boldsymbol{I}_{mn\times mn} & \boldsymbol{0}_{mn\times(l+6)n}\end{pmatrix};\ \boldsymbol{\Sigma}_{Q_2}=\begin{pmatrix}\boldsymbol{0}_{ln\times(m+4)n} & \boldsymbol{I}_{ln\times ln} & \boldsymbol{0}_{ln\times 3n}\end{pmatrix};\ \boldsymbol{\Sigma}_{\tilde{Q}_2}=\begin{pmatrix}\boldsymbol{0}_{ln\times(m+5)n} & \boldsymbol{I}_{ln\times ln} & \boldsymbol{0}_{ln\times 2n}\end{pmatrix};$$

$$\boldsymbol{\Sigma}_{\tilde{Q}_3}=\begin{pmatrix}\boldsymbol{I}_{n\times n} & \boldsymbol{0}_{n\times(m-1)n} & \boldsymbol{0}_{n\times n} & \boldsymbol{0}_{n\times n} & \boldsymbol{0}_{n\times n} & \boldsymbol{0}_{n\times n} & \boldsymbol{0}_{n\times(l+3)n}\\ \boldsymbol{0}_{n\times n} & \boldsymbol{0}_{n\times(m-1)n} & \boldsymbol{0}_{n\times n} & \boldsymbol{0}_{n\times n} & \boldsymbol{I}_{n\times n} & \boldsymbol{0}_{n\times n} & \boldsymbol{0}_{n\times(l+3)n}\\ \boldsymbol{0}_{n\times n} & \boldsymbol{0}_{n\times(m-1)n} & \boldsymbol{I}_{n\times n} & \boldsymbol{0}_{n\times n} & \boldsymbol{0}_{n\times n} & \boldsymbol{0}_{n\times n} & \boldsymbol{0}_{n\times(l+3)n}\\ \boldsymbol{0}_{n\times n} & \boldsymbol{0}_{n\times(m-1)n} & \boldsymbol{0}_{n\times n} & \boldsymbol{0}_{n\times n} & \boldsymbol{0}_{n\times n} & \boldsymbol{I}_{n\times n} & \boldsymbol{0}_{n\times(l+3)n}\end{pmatrix};$$

$$\boldsymbol{\Sigma}_{\tilde{\Lambda}_1}=\begin{pmatrix}\boldsymbol{I}_{n\times n} & \boldsymbol{0}_{n\times(m+1)n} & \boldsymbol{0}_{n\times n} & \boldsymbol{0}_{n\times(l+4)n}\\ \boldsymbol{0}_{n\times n} & \boldsymbol{0}_{n\times(m+1)n} & \boldsymbol{I}_{n\times n} & \boldsymbol{0}_{n\times(l+4)n}\end{pmatrix};\ \boldsymbol{\Sigma}_{\tilde{\Lambda}_2}=\begin{pmatrix}\boldsymbol{0}_{n\times mn} & \boldsymbol{I}_{n\times n} & \boldsymbol{0}_{n\times 2n} & \boldsymbol{0}_{n\times n} & \boldsymbol{0}_{n\times(l+3)n}\\ \boldsymbol{0}_{n\times mn} & \boldsymbol{0}_{n\times n} & \boldsymbol{0}_{n\times 2n} & \boldsymbol{I}_{n\times n} & \boldsymbol{0}_{n\times(l+3)n}\end{pmatrix};$$

$$\boldsymbol{\Sigma}_{\tilde{R}}=\begin{pmatrix}\boldsymbol{0}_{n\times n} & \boldsymbol{0}_{n\times mn} & \sqrt{\dfrac{\tau}{m}}\boldsymbol{I}_{n\times n} & \boldsymbol{0}_{n\times n} & \boldsymbol{0}_{n\times(l+2)n} & \boldsymbol{0}_{n\times n} & \boldsymbol{0}_{n\times n}\\ \sqrt{\dfrac{\tau}{m}}\boldsymbol{I}_{n\times n} & \boldsymbol{0}_{n\times mn} & \boldsymbol{0}_{n\times n} & \boldsymbol{0}_{n\times n} & \boldsymbol{0}_{n\times(l+2)n} & \boldsymbol{0}_{n\times n} & \boldsymbol{0}_{n\times n}\\ \boldsymbol{0}_{n\times n} & \boldsymbol{0}_{n\times mn} & \sqrt{\tau}\boldsymbol{I}_{n\times n} & \boldsymbol{0}_{n\times n} & \boldsymbol{0}_{n\times(l+2)n} & \boldsymbol{0}_{n\times n} & \boldsymbol{0}_{n\times n}\\ \boldsymbol{0}_{n\times n} & \boldsymbol{0}_{n\times mn} & \boldsymbol{0}_{n\times n} & \sqrt{\dfrac{r}{l}}\boldsymbol{I}_{n\times n} & \boldsymbol{0}_{n\times(l+2)n} & \boldsymbol{0}_{n\times n} & \boldsymbol{0}_{n\times n}\\ \boldsymbol{0}_{n\times n} & \boldsymbol{0}_{n\times mn} & \boldsymbol{0}_{n\times n} & \boldsymbol{0}_{n\times n} & \boldsymbol{0}_{n\times(l+2)n} & \sqrt{\dfrac{m}{\tau}}\boldsymbol{I}_{n\times n} & \boldsymbol{0}_{n\times n}\\ \boldsymbol{0}_{n\times n} & \boldsymbol{0}_{n\times mn} & \boldsymbol{0}_{n\times n} & \boldsymbol{0}_{n\times n} & \boldsymbol{0}_{n\times(l+2)n} & \boldsymbol{0}_{n\times n} & \sqrt{\dfrac{l}{r}}\boldsymbol{I}_{n\times n}\end{pmatrix};$$

$$\boldsymbol{\Sigma}_{\tilde{S}}=\begin{pmatrix}\boldsymbol{0}_{n\times(m+1)n} & \dfrac{1}{\sqrt{2}}\dfrac{\tau}{m}\boldsymbol{I}_{n\times n} & \boldsymbol{0}_{n\times n} & \boldsymbol{0}_{n\times(l+3)n} & \boldsymbol{0}_{n\times n}\\ \boldsymbol{0}_{n\times(m+1)n} & \boldsymbol{0}_{n\times n} & \dfrac{r}{\sqrt{2}l}\boldsymbol{I}_{n\times n} & \boldsymbol{0}_{n\times(l+3)n} & \boldsymbol{0}_{n\times n}\\ \boldsymbol{0}_{n\times(m+1)n} & \boldsymbol{0}_{n\times n} & \boldsymbol{0}_{n\times n} & \boldsymbol{0}_{n\times(l+3)n} & \dfrac{\sqrt{2}l}{r}\boldsymbol{I}_{n\times n}\end{pmatrix};$$

$$\tilde{\boldsymbol{\Lambda}}_1=\begin{pmatrix}-\boldsymbol{\Lambda}_1\boldsymbol{L}_3 & \boldsymbol{\Lambda}_1\boldsymbol{L}_4\\ * & -\boldsymbol{\Lambda}_1\end{pmatrix};\ \tilde{\boldsymbol{\Lambda}}_2=\begin{pmatrix}-\boldsymbol{\Lambda}_2\boldsymbol{L}_3 & \boldsymbol{\Lambda}_2\boldsymbol{L}_4\\ * & -\boldsymbol{\Lambda}_2\end{pmatrix};$$

$\tilde{\boldsymbol{Q}}_3=\mathrm{diag}(\boldsymbol{Q}_3,-\boldsymbol{Q}_3)$;$\tilde{\boldsymbol{R}}=\mathrm{diag}(\boldsymbol{R}_1,\boldsymbol{R}_2,\boldsymbol{R}_3,\boldsymbol{R}_4,-\boldsymbol{R}_2,-\boldsymbol{R}_4)$;$\tilde{\boldsymbol{S}}=\mathrm{diag}(\boldsymbol{S}_1,\boldsymbol{S}_2,-\boldsymbol{S}_2)$;$\tilde{\boldsymbol{M}}'^{\mathrm{T}}=\left(\boldsymbol{M}_1^{\mathrm{T}},\boldsymbol{M}_2^{\mathrm{T}},\boldsymbol{0}_{n\times(m+l+5)n}\right)$;

$\hat{\boldsymbol{M}}'=\left(\boldsymbol{I}_{n\times n},-\boldsymbol{I}_{n\times n},\boldsymbol{0}_{n\times(m+l+5)n}\right)$;$\tilde{\boldsymbol{N}}'^{\mathrm{T}}=\left(\boldsymbol{N}_1^{\mathrm{T}},\boldsymbol{0}_{n\times(m-1)n},\boldsymbol{N}_2^{\mathrm{T}},\boldsymbol{0}_{n\times(l+6)n}\right)$;$\hat{\boldsymbol{N}}'=\left(\boldsymbol{I}_{n\times n},\boldsymbol{0}_{n\times(m-1)n},-\boldsymbol{I}_{n\times n},\boldsymbol{0}_{n\times(l+6)n}\right)$;

$\tilde{\boldsymbol{F}}'^{\mathrm{T}}=\left(\boldsymbol{F}_1^{\mathrm{T}},\boldsymbol{0}_{n\times(m+l+4)n},\boldsymbol{F}_2^{\mathrm{T}},\boldsymbol{0}_{n\times n}\right)$;$\hat{\boldsymbol{F}}'=\left(\dfrac{\tau}{m}\boldsymbol{I}_{n\times n},\boldsymbol{0}_{n\times(m+l+4)n},-\boldsymbol{I}_{n\times n},\boldsymbol{0}_{n\times n}\right)$;$\tilde{\boldsymbol{G}}'^{\mathrm{T}}=\left(\boldsymbol{G}_1^{\mathrm{T}},\boldsymbol{0}_{n\times mn},\boldsymbol{G}_2^{\mathrm{T}},\boldsymbol{G}_3^{\mathrm{T}},\boldsymbol{G}_4^{\mathrm{T}},\boldsymbol{0}_{n\times(l+3)n}\right)$;

$\hat{\boldsymbol{G}}'=(-\boldsymbol{C},\boldsymbol{0}_{n\times mn},-\boldsymbol{I}_{n\times n},\boldsymbol{A},\boldsymbol{B},\boldsymbol{D},\boldsymbol{D},\cdots,\boldsymbol{D},\boldsymbol{0}_{n\times 3n})$。

情形2：若在系统(2)中令 $\boldsymbol{D}=\boldsymbol{0}$，则经过平衡点平移变换后，相应的系统即为在很多文献中得到广泛研究的时变时滞神经网络系统：

$$\dot{\boldsymbol{x}}(t)=-\boldsymbol{C}\boldsymbol{x}(t)+\boldsymbol{A}\boldsymbol{f}\big(\boldsymbol{x}(t)\big)+\boldsymbol{B}\boldsymbol{f}\Big(\boldsymbol{x}\big(t-\tau(t)\big)\Big)。\quad(24)$$

相应地有关于系统(24)的如下推论。

推论2：对于给定的 τ_1、τ_2、τ_3，正整数 $m\geqslant 1$，若存在对称正定阵 $\boldsymbol{P}\in\mathbb{R}^{2n\times 2n}$，$\boldsymbol{Q}_1\in\mathbb{R}^{mn\times mn}$，$\boldsymbol{Q}_3\in\mathbb{R}^{2n\times 2n}$，$\boldsymbol{Z},\boldsymbol{R}_i\in\mathbb{R}^{n\times n}\ (i=1,2,3)$，$\boldsymbol{S}_1\in\mathbb{R}^{n\times n}$，非负对角阵 $\boldsymbol{W}=\mathrm{diag}(w_1,w_2,\cdots,w_n)$，$\boldsymbol{\Delta}=\mathrm{diag}(\delta_1,\delta_2,\cdots,\delta_n)$，$\boldsymbol{\Lambda}_i=\mathrm{diag}(\lambda_{i1},\lambda_{i2},\cdots,\lambda_{in}),i=1,2$，以及任意恰当维数的矩阵 $\boldsymbol{M}_i$、$\boldsymbol{N}_i$、$\boldsymbol{F}_i$、$\boldsymbol{G}_j$（$i=1,2;j=1,2,3,4$），使如下的LMIs成立，则系统(24)的平衡点是全局渐近稳定的，

$$\begin{pmatrix}\boldsymbol{\Pi} & \sqrt{\frac{\tau}{m}}\tilde{\boldsymbol{M}}'' & \sqrt{\tau}\tilde{\boldsymbol{N}}'' & \frac{1}{\sqrt{2}}\frac{\tau}{m}\tilde{\boldsymbol{F}}'' \\ * & -\boldsymbol{R}_1 & \boldsymbol{0} & \boldsymbol{0} \\ * & * & -\boldsymbol{R}_3 & \boldsymbol{0} \\ * & * & * & -\boldsymbol{S}_1\end{pmatrix}<0，$$

式中，$\boldsymbol{\Pi}$ 的定义为：

$$\boldsymbol{\Pi}=\mathrm{sym}\Big(\boldsymbol{\Pi}_{P_1}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{\Pi}_{P_2}\Big)+\mathrm{sym}\Big(\boldsymbol{\Pi}_{w}^{\mathrm{T}}\tilde{\boldsymbol{W}}\boldsymbol{\Pi}_{\Delta}\Big)+\boldsymbol{\Pi}_{Q_1}^{\mathrm{T}}\boldsymbol{Q}_1\boldsymbol{\Pi}_{Q_1}-\boldsymbol{\Pi}_{Q'}^{\mathrm{T}}\boldsymbol{Q}'_1\boldsymbol{\Pi}_{Q'}+\boldsymbol{\Pi}_{\tilde{Q}_3}^{\mathrm{T}}\tilde{\boldsymbol{Q}}_3\boldsymbol{\Pi}_{\tilde{Q}_3}+\boldsymbol{\Pi}_{\tilde{Z}}^{\mathrm{T}}\tilde{\boldsymbol{Z}}\boldsymbol{\Pi}_{\tilde{Z}}+\boldsymbol{\Pi}_{\tilde{R}}^{\mathrm{T}}\tilde{\boldsymbol{R}}\boldsymbol{\Pi}_{\tilde{R}}+$$
$$\boldsymbol{\Pi}_{\tilde{S}}^{\mathrm{T}}\tilde{\boldsymbol{S}}\boldsymbol{\Pi}_{\tilde{S}}+\mathrm{sym}\Big(\tilde{\boldsymbol{M}}''\hat{\boldsymbol{M}}''\Big)+\mathrm{sym}\Big(\tilde{\boldsymbol{N}}''\hat{\boldsymbol{N}}''\Big)+\mathrm{sym}\Big(\tilde{\boldsymbol{F}}''\hat{\boldsymbol{F}}''\Big)+\mathrm{sym}\Big(\tilde{\boldsymbol{G}}''\hat{\boldsymbol{G}}''\Big)+\boldsymbol{\Pi}_{\tilde{\Lambda}_1}^{\mathrm{T}}\tilde{\boldsymbol{\Lambda}}_1\boldsymbol{\Pi}_{\tilde{\Lambda}_1}+\boldsymbol{\Pi}_{\tilde{\Lambda}_2}^{\mathrm{T}}\tilde{\boldsymbol{\Lambda}}_2\boldsymbol{\Pi}_{\tilde{\Lambda}_2}。\quad(25)$$

式(25)中：

$$\boldsymbol{\Pi}_{P_1}=\begin{pmatrix}\boldsymbol{I}_{n\times n} & \boldsymbol{0}_{n\times(m+6)n} & \boldsymbol{0}_{n\times n}\\ \boldsymbol{0}_{n\times n} & \boldsymbol{0}_{n\times(m+6)n} & \boldsymbol{I}_{n\times n}\end{pmatrix}；\boldsymbol{\Pi}_{P_2}=\begin{pmatrix}\boldsymbol{0}_{n\times n} & \boldsymbol{0}_{n\times n} & \boldsymbol{0}_{n\times(m+1)n} & \boldsymbol{I}_{n\times n} & \boldsymbol{0}_{n\times 4n}\\ \boldsymbol{I}_{n\times n} & -\boldsymbol{I}_{n\times n} & \boldsymbol{0}_{n\times(m+1)n} & \boldsymbol{0}_{n\times n} & \boldsymbol{0}_{n\times 4n}\end{pmatrix}；\boldsymbol{\Pi}_{W}=\begin{pmatrix}-\boldsymbol{L}_1 & \boldsymbol{0}_{n\times(m+3)n} & \boldsymbol{I}_{n\times n} & \boldsymbol{0}_{n\times 3n}\\ \boldsymbol{L}_2 & \boldsymbol{0}_{n\times(m+3)n} & -\boldsymbol{I}_{n\times n} & \boldsymbol{0}_{n\times 3n}\end{pmatrix}；$$

$$\boldsymbol{\Pi}_{\Delta}=\begin{pmatrix}\boldsymbol{0}_{n\times(m+3)n} & \boldsymbol{I}_{n\times n} & \boldsymbol{0}_{n\times 4n}\\ \boldsymbol{0}_{n\times(m+3)n} & \boldsymbol{I}_{n\times n} & \boldsymbol{0}_{n\times 4n}\end{pmatrix}；\tilde{\boldsymbol{W}}=\begin{pmatrix}\boldsymbol{W} & \\ & \boldsymbol{\Delta}\end{pmatrix}；\boldsymbol{\Pi}_{\tilde{Z}}=\begin{pmatrix}\boldsymbol{0}_{n\times mn} & \boldsymbol{I}_{n\times n} & \boldsymbol{0}_{n\times n} & \boldsymbol{0}_{n\times n} & \boldsymbol{0}_{n\times 5n}\\ \boldsymbol{0}_{n\times mn} & \boldsymbol{0}_{n\times n} & \boldsymbol{0}_{n\times n} & \boldsymbol{I}_{n\times n} & \boldsymbol{0}_{n\times 5n}\end{pmatrix}；$$

$$\boldsymbol{\Pi}_{Q_1}=\begin{pmatrix}\boldsymbol{I}_{mn\times mn} & \boldsymbol{0}_{mn\times 8n}\end{pmatrix}；\boldsymbol{\Pi}_{Q'_1}=\begin{pmatrix}\boldsymbol{0}_{mn\times n} & \boldsymbol{I}_{mn\times mn} & \boldsymbol{0}_{mn\times 7n}\end{pmatrix}；$$

$$\boldsymbol{\Pi}_{\tilde{Q}_3}=\begin{pmatrix}\boldsymbol{0}_{n\times mn} & \boldsymbol{I}_{n\times n} & \boldsymbol{0}_{n\times n} & \boldsymbol{0}_{n\times 3n} & \boldsymbol{0}_{n\times n} & \boldsymbol{0}_{n\times n} & \boldsymbol{0}_{n\times n}\\ \boldsymbol{0}_{n\times mn} & \boldsymbol{0}_{n\times n} & \boldsymbol{0}_{n\times n} & \boldsymbol{0}_{n\times 3n} & \boldsymbol{I}_{n\times n} & \boldsymbol{0}_{n\times n} & \boldsymbol{0}_{n\times n}\\ \boldsymbol{0}_{n\times mn} & \boldsymbol{0}_{n\times n} & \boldsymbol{I}_{n\times n} & \boldsymbol{0}_{n\times 3n} & \boldsymbol{0}_{n\times n} & \boldsymbol{0}_{n\times n} & \boldsymbol{0}_{n\times n}\\ \boldsymbol{0}_{n\times mn} & \boldsymbol{0}_{n\times n} & \boldsymbol{0}_{n\times n} & \boldsymbol{0}_{n\times 3n} & \boldsymbol{0}_{n\times n} & \boldsymbol{I}_{n\times n} & \boldsymbol{0}_{n\times n}\end{pmatrix}；\boldsymbol{\Pi}_{\tilde{\Lambda}_1}=\begin{pmatrix}\boldsymbol{I}_{n\times n} & \boldsymbol{0}_{n\times(m+3)n} & \boldsymbol{0}_{n\times n} & \boldsymbol{0}_{n\times 3n}\\ \boldsymbol{0}_{n\times n} & \boldsymbol{0}_{n\times(m+3)n} & \boldsymbol{I}_{n\times n} & \boldsymbol{0}_{n\times 3n}\end{pmatrix}；$$

$$\boldsymbol{\Pi}_{\tilde{\Lambda}_2}=\begin{pmatrix}\boldsymbol{0}_{n\times mn} & \boldsymbol{I}_{n\times n} & \boldsymbol{0}_{n\times 4n} & \boldsymbol{0}_{n\times n} & \boldsymbol{0}_{n\times 2n}\\ \boldsymbol{0}_{n\times mn} & \boldsymbol{0}_{n\times n} & \boldsymbol{0}_{n\times 4n} & \boldsymbol{I}_{n\times n} & \boldsymbol{0}_{n\times 2n}\end{pmatrix}；\boldsymbol{\Sigma}_{\tilde{R}}=\begin{pmatrix}\boldsymbol{0}_{n\times n} & \boldsymbol{0}_{n\times(m+2)n} & \sqrt{\frac{\tau_1}{m}}\boldsymbol{I}_{n\times n} & \boldsymbol{0}_{n\times 3n} & \boldsymbol{0}_{n\times n}\\ \sqrt{\frac{\tau_1}{m}}\boldsymbol{I}_{n\times n} & \boldsymbol{0}_{n\times(m+2)n} & \boldsymbol{0}_{n\times n} & \boldsymbol{0}_{n\times 3n} & \boldsymbol{0}_{n\times n}\\ \boldsymbol{0}_{n\times n} & \boldsymbol{0}_{n\times(m+2)n} & \boldsymbol{0}_{n\times n} & \boldsymbol{0}_{n\times 3n} & \sqrt{\frac{m}{\tau_1}}\boldsymbol{I}_{n\times n}\\ \boldsymbol{0}_{n\times n} & \boldsymbol{0}_{n\times(m+2)n} & \sqrt{\tau_2-\tau_1}\boldsymbol{I}_{n\times n} & \boldsymbol{0}_{n\times 3n} & \boldsymbol{0}_{n\times n}\end{pmatrix}；$$

$$\boldsymbol{\Pi}_{S}=\left(\boldsymbol{0}_{n\times(m+3)n},\frac{1}{\sqrt{2}}\frac{\tau_1}{m}\boldsymbol{I}_{n\times n},\boldsymbol{0}_{n\times 4n}\right)；\tilde{\boldsymbol{\Lambda}}_1=\begin{pmatrix}-\boldsymbol{\Lambda}_1\boldsymbol{L}_3 & \boldsymbol{\Lambda}_1\boldsymbol{L}_4\\ * & -\boldsymbol{\Lambda}_1\end{pmatrix}；\tilde{\boldsymbol{\Lambda}}_2=\begin{pmatrix}-\boldsymbol{\Lambda}_2\boldsymbol{L}_3 & \boldsymbol{\Lambda}_2\boldsymbol{L}_4\\ * & -\boldsymbol{\Lambda}_2\end{pmatrix}；$$

$$\tilde{\boldsymbol{Q}}_3=\mathrm{diag}\big(\boldsymbol{Q}_3,-(1-\tau_3)\boldsymbol{Q}_3\big)；\tilde{\boldsymbol{Z}}=\mathrm{diag}(\boldsymbol{Z},-\boldsymbol{Z})；\tilde{\boldsymbol{R}}=\mathrm{diag}(\boldsymbol{R}_1,\boldsymbol{R}_2,-\boldsymbol{R}_2,\boldsymbol{R}_3)；$$

$$\tilde{\boldsymbol{M}}''^{\mathrm{T}}=\left(\boldsymbol{M}_1^{\mathrm{T}},\boldsymbol{M}_2^{\mathrm{T}},\boldsymbol{0}_{n\times(m+6)n}\right)；\hat{\boldsymbol{M}}''=\left(\boldsymbol{I}_{n\times n},-\boldsymbol{I}_{n\times n},\boldsymbol{0}_{n\times(m+6)n}\right)；\tilde{\boldsymbol{N}}''^{\mathrm{T}}=\left(\boldsymbol{0}_{n\times mn},\boldsymbol{N}_1^{\mathrm{T}},\boldsymbol{0}_{n\times n},\boldsymbol{N}_2^{\mathrm{T}},\boldsymbol{0}_{n\times 5n}\right)；$$

$$\hat{\boldsymbol{N}}''=\left(\boldsymbol{0}_{n\times mn},\boldsymbol{I}_{n\times n},\boldsymbol{0}_{n\times n},-\boldsymbol{I}_{n\times n},\boldsymbol{0}_{n\times 5n}\right)；\tilde{\boldsymbol{F}}''^{\mathrm{T}}=\left(\boldsymbol{F}_1^{\mathrm{T}},\boldsymbol{0}_{n\times(m+6)n},\boldsymbol{F}_2^{\mathrm{T}}\right)；\hat{\boldsymbol{F}}''=\left(\frac{\tau_1}{m}\boldsymbol{I}_{n\times n},\boldsymbol{0}_{n\times(m+6)n},-\boldsymbol{I}_{n\times n}\right)；$$

$$\tilde{\boldsymbol{G}}''^{\mathrm{T}}=\left(\boldsymbol{G}_1^{\mathrm{T}},\boldsymbol{0}_{n\times(m+2)n},\boldsymbol{G}_2^{\mathrm{T}},\boldsymbol{G}_3^{\mathrm{T}},\boldsymbol{0}_{n\times 3n}\right)；\hat{\boldsymbol{G}}''=\left(-\boldsymbol{C},\boldsymbol{0}_{n\times(m+2)n},-\boldsymbol{I}_{n\times n},\boldsymbol{A},\boldsymbol{0}_{n\times n},\boldsymbol{B},\boldsymbol{0}_{n\times n}\right)。$$

3 数值实例

这里将给出2个数值实例，用本文方法计算结果，并与相关文献结果进行比较。

例1：考虑文献[18,20]给出的2阶时滞神经网络系统，相关参数如下：

$$\boldsymbol{C}=\boldsymbol{I},\ \boldsymbol{A}=\begin{pmatrix}-0.1 & 0.1\\ 0.1 & -0.1\end{pmatrix},\ \boldsymbol{B}=\begin{pmatrix}-0.1 & 0.2\\ 0.2 & 0.1\end{pmatrix},$$

$$\boldsymbol{D}=\begin{pmatrix}-1.8 & 0.7\\ -0.4 & -0.6\end{pmatrix},\ l_1^-=l_2^-=0,\ l_1^+=l_2^+=0.2。$$

根据由文献[20]稳定性判据得到的时滞最大允许上界为6.827 9；当 m、l 分别都取相等的1、2、3、4时，文献[18]给出的相应结果分别为5.514 7、7.236 8、8.582 6和9.724 5。而用本文推论1的结论，当 m、l 分别都取相等的1、2、3、4时，相应的结果则为6.846 1、7.257 9、8.697 4和9.752 3。显然，本文结果相较于文

献[18,20]的结果具有更低的保守性。

例2:考虑文献[21-24]中给出的2阶时滞神经网络系统,相关参数如下:

$$C=\mathrm{diag}(2,2),\ A=\begin{pmatrix}1 & 1\\ -1 & -1\end{pmatrix},\ B=\begin{pmatrix}0.88 & 1\\ 1 & 1\end{pmatrix},$$

$$l_1^-=l_2^-=0,\ l_1^+=0.4,\ l_2^+=0.8。$$

为与已有文献结果进行比较,设 $\tau_1=0$,$\tau_3=0.6$,则 τ_2 的MAUB分别为2.921 9[24]、2.933 4[23]。而用本文推论2的结论,当时滞分割数 m 分别取1、2、3、4时,τ_2 的MAUB分别为2.965 1、2.997 4、3.011 2和3.120 7。

尽管时滞分割能降低系统稳定性条件的保守性,但必须指出的是,随着时滞分割数 m 或 r 的增加,保守性的降低将越来越不明显,而计算的复杂程度却越来越高。在实际应用中,应当兼顾保守性和计算复杂度,尽量选取适当的时滞分割数。

4 结语

本文基于时滞分割技术研究了一类同时具有时变时滞和分布时滞的神经网络系统的全局稳定性,通过对时变时滞函数的下界和定常分布时滞进行时滞分割并构造一个新的增广Lyapunov-Krasovskii泛函,获得系统一个改进的时滞相依全局渐近稳定性充分条件,时滞分割是降低稳定性条件保守性的关键因素。保守性随着时滞分割数目的增加而降低,但降低幅度越来越小。稳定性条件是以LMIs给出的,便于使用标准的数字软件包加以检验,数值实例清晰表明时滞分割技术可以有效地降低稳定性条件的保守性。

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A Delay-Dependent Global Stability Analysis of Neural Networks With Time-Varying and Distributed Delays

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Abstract: In this paper, together with an improved augmented Lyapunov-Krasovskii functional and delay partitioning, which was applied to both the lower bound of time-varying and the constant distributed delays, a novel delay-dependent sufficient condition was obtained for guaranteeing a class of neural networks with and time-varying and distributed delays to be global asymptotically stable, which was in form of LMIs. Examples were given to illustrate that the results in this paper is more efficiently and less conservative than some existing ones.

Key words: distributed delays; delay partitioning; delay dependent; global asymptotically stable

(上接第327页)

Research on Rocket Trajectory Based on Particle Swarm Optimization Algorithm

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Abstract: Aiming at the problem of gliding and extending the range of guided rockets, a solution with angle of attack as the optimization variable was proposed in this paper. Based on the four-degree-of-freedom model of the rockets, the rocket-range optimization model was proposed based on the particle swarm optimization algorithm, and the constraints were established. The optimization simulation of a guided rocket trajectory show that the control of the angle of attack of the rocket can effectively increase the range of the rocket. For the rocket gliding range problem, the angle of attack control method proposed in this paper is feasible.

Key words: gliding extened range; particle swarm optimization; angle of attack